

Which Part Is the Hamiltonian Is a Convention

— Complete Positivity Is a Demand About Something One Is Not Touching: the Fence Around the Lindblad Equation, and the Two Destinations of Decoherence — [Paper 138]

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— *From thinking to syncing.* —

ABSTRACT

The Lindblad equation writes the evolution of an open system as **a unitary part plus a dissipative part**. **This paper writes that the split is not unique, what fence makes it unique, and that the word “decoherence” names two different destinations. NO NEW MATHEMATICAL THEOREM AND NO NEW PHYSICAL LAW IS CLAIMED.** **First, one of the two cores. The split is not unique.** Shifting a jump operator as $L \rightarrow L + cI$ and the Hamiltonian by $\Delta H = \frac{i}{2}(cL^\dagger - \bar{c}L)$ leaves the difference of the two generators exactly the zero matrix (confirmed here symbolically). The same evolution is described by different Hamiltonians (Section 2). **Second, there is a fence that makes it unique.** Imposing $\text{tr}(L) = 0$ gives $\text{tr}(L') = 2c$ and forces $c = 0$. **Traceless jump operators, plus dropping the additive constant in H , make the decomposition unique. Uniqueness lives only inside the fence (Papers 127, 133) (Section 3).** **Third, the other core. Complete positivity is a demand about something one is not touching.** The transpose map is **positive**: over 20,000 random qubit states this paper finds the smallest eigenvalue after transposition to be at least 3.7×10^{-5} . Yet **transposing only one half of a Bell state produces the eigenvalue -0.5 . Nothing breaks while one looks at the system alone; something breaks when it is entangled with a spectator. That is why the Lindblad form looks the way it does (Section 4).** **Fourth, $T_2 \leq 2T_1$ is forced.** Since $1/T_2 = 1/(2T_1) + 1/T_\phi$, equality holds only when there is no pure dephasing. **One word, “decay”, names two rates: energy relaxation and phase randomisation (Section 5).** **Fifth, the statement that decoherence destroys purity is false in general.** Starting from an equal superposition, **amplitude damping takes the purity down to 0.875 at $t = \ln 2$ and then back to 1**, because the fixed point at zero temperature is a pure state, the ground state. **Pure dephasing sends it monotonically to 0.5. What decides the destination is the fixed point of the map, not that the map is dissipative (Section 6).**

Scope of this paper: NO NEW MATHEMATICAL THEOREM AND NO NEW PHYSICAL LAW IS CLAIMED.

Lindblad (1976), Gorini–Kossakowski–Sudarshan (1976), Stinespring's dilation theorem, the Peres–Horodecki criterion and the relation between T_1 and T_2 are **all standard**. What this paper adds is only **exhibiting the gauge freedom and showing the difference of generators to be zero, writing the fence that makes the split unique, demonstrating complete positivity as a demand about a spectator, and separating the two destinations of decoherence.**

1. Notation and position

Write the evolution of a density matrix ρ as

$$\dot{\rho} = -i[H, \rho] + \sum_k \left(L_k \rho L_k^\dagger - \frac{1}{2} \{L_k^\dagger L_k, \rho\} \right) \quad (1)$$

with $\hbar = 1$, calling the first term **unitary** and the second **dissipative**.

Position: existing theorems are laid side by side and their reach written down. No new theorem is claimed. Paper 118 said that dt and dS are of different logical type. Equation (1) looks like the two of them set side by side in one line — this paper asks how far that arrangement is unique.

2. The split is not unique

Verification 2.1 (a gauge freedom): For a single jump operator, move

$$L \longrightarrow L + cI, \quad H \longrightarrow H + \frac{i}{2} (cL^\dagger - \bar{c}L)$$

together. For $L = \sqrt{\gamma} \sigma_-$ and $H = 0$ this paper confirms symbolically that ΔH is **Hermitian** and that the difference of the two generators is **exactly the zero matrix**. (QED)

Observation 2.2 (what follows)

The same evolution can be written with different Hamiltonians and different jump operators. So the statement “the Hamiltonian of this system is such-and-such” has no content by itself: which decomposition was taken must be said.

Observation 2.3 (why)

Adding the identity to a jump operator produces a commutator $\frac{1}{2}[\bar{c}L - cL^\dagger, \rho]$ inside the dissipative term. A commutator has the same shape as the unitary term, so it can be moved from one side to the other. One word, “Hamiltonian”, is naming a quantity that depends on our choice (Paper 95).

3. The fence that makes it unique

Observation 3.1

This paper computes $\text{tr}(L) = 0$ and $\text{tr}(L + cI) = 2c$. Requiring traceless jump operators therefore forces $c = 0$.

Observation 3.2 (the shape of the fence)

what is imposed	the decomposition is
nothing	not unique (the freedom in c)
traceless jump operators	unique up to a constant in H
and $\text{tr}(H) = 0$ as well	completely unique

The same shape as Papers 127 and 133: uniqueness lives only inside a fence, and must not be accepted without counting it. Here the fence is tracelessness.

Observation 3.3 (what this means in practice)

What an experiment measures is the generator, not its decomposition. A quantity such as “the size of the Lamb shift” becomes a number only once the traceless convention is adopted.

4. Complete positivity is a demand about a spectator

Established Fact 4.1: A map is **positive** if it sends density matrices to density matrices. It is **completely positive** if the map extended by the identity on a spectator of any size is still positive.

Verification 4.2 (transposition is positive): For the transpose map on a single qubit this paper confirms that **over 20,000 random states the smallest eigenvalue after transposition is positive** (minimum 3.7×10^{-5}). **Nothing breaks so long as one looks at the system alone.**

Verification 4.3 (but it is not completely positive): Transposing **only one half** of a Bell state, this paper obtains eigenvalues

$$\{-0.5, 0.5, 0.5, 0.5\},$$

one of which is **negative**. (QED)

Observation 4.4 (this is the point)

Complete positivity is a demand about something one is not touching. It cannot be checked by looking inside the system. The Lindblad form has the shape of (1) as a consequence of that demand (Stinespring's dilation theorem).

And this is an instance of the corpus's distinct roots: “positive” and “completely positive” look like one family of words, and neither implies the other automatically. The separator is whether a spectator is present, and the transpose map demonstrates it.

5. $T_2 \leq 2T_1$

Observation 5.1

With amplitude damping at rate Γ_1 and pure dephasing at Γ_φ , the populations decay at Γ_1 and the coherence at $\Gamma_1/2 + \Gamma_\varphi$, so

$$\frac{1}{T_2} = \frac{1}{2T_1} + \frac{1}{T_\varphi}, \quad \text{whence} \quad T_2 \leq 2T_1.$$

Verification 5.2

T_1	T_φ	T_2
1.0	∞	2.0000 (at the bound)
1.0	10.0	1.6667
1.0	1.0	0.6667
1.0	0.1	0.0952

(QED)

Observation 5.3 (one word, two rates)

“Decay” names energy relaxation and phase randomisation, which are different things. Only their combination is observed, and T_2 alone does not reveal the breakdown. If $T_2 = 2T_1$ is met, one learns that there is no pure dephasing: touching the bound is itself information.

6. Two decoherences, two destinations

Verification 6.1: The purity $\text{tr}(\rho^2)$ starting from an equal superposition.

t	amplitude damping	pure dephasing
0.0	1.000000	1.000000
0.5	0.880674	0.683940
1.0	0.883728	0.567668
2.0	0.941490	0.509158
10.0	0.999977	0.500000

Under amplitude damping the purity falls to 0.875 at $t = 0.693$ and then returns to 1. Under pure dephasing it falls monotonically to 0.5. (QED)

Observation 6.2 (correcting the phrase)

“Decoherence destroys purity” is not right in general. Amplitude damping at zero temperature has a pure state, the ground state, as its fixed point, so the purity comes back.

Observation 6.3 (what decides the destination)

Being dissipative does not by itself decide where the system ends. The fixed point of the map does. Within the one frame of “an open system” there are processes ending pure and processes ending mixed.

Observation 6.4 (against Paper 135)

In Paper 135 a symplectic map had determinant exactly 1 and the volume was preserved exactly. Here it is not — and that is not a defect of an integrator but the physics being described. Two things called “not preserved”, numerical error and physical dissipation, are distinct roots.

7. Conclusion

1. The split is not unique (Ver. 2.1). Under $L \rightarrow L + cI$ and $H \rightarrow H + \frac{i}{2}(cL^\dagger - \bar{c}L)$ the difference of the generators is exactly the zero matrix. “The Hamiltonian of this system” has no content without naming the decomposition.
2. The fence is tracelessness (Obs. 3.1). Imposing $\text{tr}(L) = 0$ forces $c = 0$. Uniqueness lives only inside the fence (Papers 127, 133).
3. Complete positivity is a demand about a spectator (Ver. 4.2, 4.3). Transposition is positive (minimum eigenvalue 3.7×10^{-5} over 20,000 states) yet gives -0.5 on half of a Bell state. It cannot be checked from inside the system.
4. $T_2 \leq 2T_1$ is forced (Ver. 5.2). “Decay” names two rates, and touching the bound is itself information.
5. “Decoherence destroys purity” is false in general (Ver. 6.1). Amplitude damping dips to 0.875 and returns to 1; pure dephasing falls monotonically to 0.5. The fixed point decides.
6. Two things called “not preserved” are distinct roots (Obs. 6.4): the numerical error of Paper 135 and the physical dissipation here.

Closing: In writing the equation of an open system, the line between unitary and dissipative is drawn by us. And the shape of the equation itself comes from the demand that nothing break in a system we are not touching. Care for what is not being looked at is what fixes the form of the equation for what is.

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