

A Limit Exists Because Two Forces Carry the Same Exponent

— The Three Layers of the Chandrasekhar Mass: Dimension Gives the Form, Lane–Emden Gives the Number,
Composition Gives μ_e — [Paper 141]

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— *From thinking to syncing.* —

ABSTRACT

A white dwarf has an upper limit of mass. **This paper writes why it appears as a limit at all, and which three layers the number is made of. NO NEW MATHEMATICAL THEOREM AND NO NEW PHYSICAL LAW IS CLAIMED.** **First, and this is the core. A limit exists because two forces carry the same power of the radius.** Writing hydrostatic balance by scaling with $P = K\rho^\Gamma$, the mass depends on the radius as R^{-3} at $\Gamma = 5/3$, R^{-1} at $\Gamma = 3/2$, and exactly R^0 at $\Gamma = 4/3$ (confirmed here symbolically). **For an ultra-relativistic degenerate gas, gravity and pressure enter with the same power of R , so the contest is settled by the coefficient alone, that is, by a single mass (Section 2). Second, the number comes from the Lane–Emden equation.** Integrating $\theta'' + \frac{2}{\xi}\theta' + \theta^3 = 0$ for $n = 3$ by fourth-order Runge–Kutta at step 10^{-6} , this paper obtains $\xi_1 = 6.89685$ and $-\xi^2\theta'|_{\xi_1} = 2.01824$ (computed here, not quoted) (Section 3). **Third, the limit is the Planck mass cubed over the nucleon mass squared.** Assembled, it gives 2.854×10^{30} kg at $\mu_e = 2$, that is **1.4350 solar masses**, and dividing M_{ch} by $M_{\text{Pl}}^3/(\mu_e m_H)^2$ gives **3.097972**. **Dimensional analysis gives the form and Lane–Emden gives the number of order one (Papers 44, 116) (Section 4). Fourth, the famous “1.4 solar masses” contains a choice of composition.** Varying μ_e , the number of nucleons per electron, gives **5.74 for hydrogen, 1.435 for helium, carbon or oxygen, and 1.24 for iron.** **The number moves with the chemistry (Section 5). Fifth, there are three layers: the form (dimensional analysis, forced), the number of order one (Lane–Emden, computed), and μ_e (composition, an input). The same three-layer shape as Paper 140 (Section 6).**

Scope of this paper: NO NEW MATHEMATICAL THEOREM AND NO NEW PHYSICAL LAW IS CLAIMED.

Chandrasekhar (1931), the Lane–Emden equation and the ultra-relativistic degeneracy pressure are **all standard**. What this paper adds is only **writing why the bound falls on the mass rather than the radius, as a coincidence of exponents, integrating the Lane–Emden constants here, writing the limit as the Planck mass cubed over the nucleon mass squared, and separating the three layers. The leading-order value is not the observed bound (Coulomb corrections, general relativity and inverse beta decay all lower it).**

1. Notation and position

Write the equation of state as $P = K\rho^\Gamma$, with polytropic index n and $\Gamma = 1 + 1/n$. A non-relativistic degenerate electron gas has $\Gamma = 5/3$; an ultra-relativistic one has $\Gamma = 4/3$.

Position: existing results are laid out and the number taken apart. No new theorem is claimed. Paper 28 said that **an exponent is not a dimension**, and Paper 116 that **the dimension stands in the denominator**. This paper turns both on the Chandrasekhar mass.

2. A limit exists because the exponents coincide

Verification 2.1 (balance written by scaling): With $\rho \sim M/R^3$, writing hydrostatic balance as

$$\frac{K\rho^\Gamma}{R} \sim \frac{GM\rho}{R^2}$$

and solving for M , symbolic computation here gives

Γ	the form of M	power of R
5/3	$K^3/(G^3R^3)$	-3
3/2	$K^2/(G^2R)$	-1
4/3	$K^{3/2}/G^{3/2}$	0

(QED)

Observation 2.2 (this is what a limit is)

At $\Gamma = 4/3$ the mass does not depend on the radius, because gravity and pressure enter with the same power of R .

So the contest is settled by the coefficient alone. At $\Gamma = 5/3$ every mass finds a radius that balances it: compress and pressure wins, expand and gravity wins. At $\Gamma = 4/3$ there is no such arbiter. Below one value of the mass every radius is supported, above it none is.

Observation 2.3 (in the corpus's idiom)

Paper 117 wrote that one root of scale invariance is **exponents lining up**. Here the two terms' exponents line up, the balance loses its ability to fix a radius, and a mass is fixed instead. Equal exponents are often the sign that something is not determined — and next to what is not determined, something else becomes determined.

3. The number comes from the Lane–Emden equation

Verification 3.1: The Lane–Emden equation for $n = 3$,

$$\theta'' + \frac{2}{\xi}\theta' + \theta^3 = 0, \quad \theta(0) = 1, \theta'(0) = 0,$$

was integrated by fourth-order Runge–Kutta at step 10^{-6} , the singularity at the origin being avoided by a series start. The result is

$$\xi_1 = 6.89685, \quad -\xi^2\theta'|_{\xi_1} = 2.01824.$$

(QED)

Observation 3.2

Neither number follows from dimensional analysis. They appear only on solving the differential equation. What is usually called “a number of order one” is generally this (Paper 101).

4. The limit is the Planck mass cubed over the nucleon mass squared

Verification 4.1: Taking the ultra-relativistic degeneracy coefficient $K = \frac{hc}{8} \left(\frac{3}{\pi}\right)^{1/3} (\mu_e m_H)^{-4/3}$ and the mass of an $n = 3$ polytrope as $M = 4\pi M_3 \left(\frac{K}{\pi G}\right)^{3/2}$, this paper obtains

μ_e	M_{ch} (kg)	in solar masses
2.00	2.854031×10^{30}	1.4350
1.00	1.141612×10^{31}	5.7399

(QED)

Verification 4.2 (identifying the form): With $M_{\text{Pl}} = \sqrt{\hbar c/G} = 2.176434 \times 10^{-8}$ kg, this paper obtains

$$\frac{M_{\text{ch}}}{M_{\text{Pl}}^3/(\mu_e m_H)^2} = 3.097972.$$

(QED)

Observation 4.3

The limit is the Planck mass cubed over the nucleon mass squared, up to a number of order one. The exponents 3 and 2 are decided by dimension (Paper 116), and μ_e stands in the denominator — the shape Paper 116 wrote of dimensions, appearing here for composition.

Observation 4.4 (a consequence of gravity being weak)

$M_{\text{Pl}}/m_H \approx 1.3 \times 10^{19}$. The limit is a macroscopic mass because that ratio is enormous. As Paper 10 said, only dimensionless quantities can be tuned, and the dimensionless quantity here is M_{Pl}/m_H .

5. The famous 1.4 contains a choice of composition

Verification 5.1: μ_e is the number of nucleons per electron.

composition	μ_e	in solar masses
hydrogen	1.000	5.7399
helium, carbon, oxygen	2.000	1.4350
iron-56	2.154	1.2373

(QED)

Observation 5.2

“The Chandrasekhar mass is 1.4 solar masses” is a statement about carbon–oxygen white dwarfs, not a constant of nature. For hydrogen it would be 5.74.

Observation 5.3 (why two)

$\mu_e = 2$ restates the fact that **nuclei have equal numbers of protons and neutrons**. The number 2 comes from nuclear physics, from neither gravity nor degeneracy. As Paper 53 said of the exponent 2, the same number can arrive from a different place — the 2 here is a ratio of nucleons to electrons.

6. The three layers

Observation 6.1

what	what fixes it	standing
the form $M \sim (\hbar c/G)^{3/2}/(\mu_e m_H)^2$	dimensional analysis	forced
the number of order one (3.098)	the Lane–Emden solution	computed
μ_e	the star's composition	an input

Observation 6.2 (the same shape as Paper 140)

Paper 140 wrote three layers of **geometry (forced)**, **dynamics (measured)**, **phase (convention)**. Here they are **forced**, **computed** and **supplied**. What is the same is that a dichotomy does not suffice.

Observation 6.3 (and it is a leading-order value)

1.435 is the leading-order value, not the observed bound. Coulomb corrections, general-relativistic effects and inverse beta decay all lower it. “1.4” must not be handled as a constant of nature: it is an approximate value, a compositional value, and a value before corrections.

7. Conclusion

1. A limit exists because the exponents coincide (Ver. 2.1). M depends on R as R^{-3} at $\Gamma = 5/3$ and as R^0 at $\Gamma = 4/3$. At $\Gamma = 4/3$ there is no arbiter.

2. **The number of order one comes from Lane–Emden** (Ver. 3.1): $\xi_1 = 6.89685$, $M_3 = 2.01824$, integrated here.
3. **The limit is the Planck mass cubed over the nucleon mass squared** (Ver. 4.2), the ratio being **3.097972**. The exponents **3** and **2** are decided by dimension (Paper 116).
4. **The famous 1.4 contains a composition** (Ver. 5.1): **5.74 for hydrogen, 1.24 for iron**. $\mu_e = 2$ comes from nuclear physics.
5. **There are three layers** (Obs. 6.1): **form (forced), number (computed), μ_e (supplied)** — the same three-layer shape as Paper 140.
6. **1.435 is a leading-order value** (Obs. 6.3), and every correction lowers it.

Closing: Where the radius stops being determined, a mass becomes determined. When the exponents of two terms line up, the balance can no longer choose a length, and it chooses a single mass instead. A limit is the other face of an indeterminacy.

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