

Move It, and Only the 2π Survives

— Confirming That Unruh's 2π and Hawking's Share One Root, and Separating Out the 4 of 8π as Peculiar to Schwarzschild — [Paper 142]

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— *From thinking to syncing.* —

ABSTRACT

Paper 52 stated that the 8π of the Hawking temperature is $4 \times 2\pi$, that this 2π is a **periodicity** and not a solid angle, and that it **shares its root with the 2π of the Unruh effect. This paper proves that, and then tests the decomposition by moving the black hole. NO NEW MATHEMATICAL THEOREM AND NO NEW PHYSICAL LAW IS CLAIMED. First, the 2π is a smoothness condition.** The Euclidean Rindler metric is, near the origin, a **plane in polar coordinates, and the condition for having no conical singularity is that the angle have period exactly 2π .** A period of π or of 4π leaves a deficit (checked here). **So the 2π of $T = a/(2\pi)$ is smoothness, not a solid angle (Section 2). Second, the 4 lives in the surface gravity.** For Schwarzschild, $\kappa = f'(r_h)/2 = 1/(4M)$ and hence $T = \kappa/(2\pi) = 1/(8\pi M)$ (computed here symbolically). **The 4 of $8\pi = 2\pi \times 4$ comes from the relation between surface gravity and mass (Section 3). Third, and this is the core. Move the black hole and the 4 moves while the 2π does not.** For Reissner–Nordström the coefficient is $1/(8\pi) = 0.039789$ at $Q/M = 0$, 0.033648 at 0.9 , 0.017243 at 0.99 and **zero** at extremality; Kerr behaves the same way. **The 8π is not universal. What is universal is $T = \kappa/(2\pi)$ (Section 4). Fourth, near the horizon the local temperature IS the Unruh temperature.** The ratio of the Tolman-shifted local temperature to the proper acceleration of a static observer runs $0.16235396 \rightarrow 0.15915495$ as $r/2M - 1$ falls from 10^{-2} to 10^{-8} , **agreeing with $1/(2\pi) = 0.15915494$ to eight digits (Section 5). Fifth, they are nevertheless not the same phenomenon.** The Unruh effect needs **no horizon of finite area, no evaporation, and no gravity. One root for the 2π ; different physics around it (Section 6).**

Scope of this paper: NO NEW MATHEMATICAL THEOREM AND NO NEW PHYSICAL LAW IS CLAIMED.

Unruh (1976), Hawking (1975), the Gibbons–Hawking Euclidean method and the relation between surface gravity and temperature are **all standard**. What this paper adds is only **demonstrating by a conical-deficit computation the shared root asserted in Paper 52, testing the decomposition by charging and spinning the hole, and confirming that the near-horizon ratio converges to $1/(2\pi)$ to eight digits. This is a paper that confirms a SHARED root rather than separating roots — for the corpus, that confirmation is part of the same roll-call.**

1. Notation and position

Set $G = c = \hbar = k_B = 1$. The Unruh temperature is $T = a/(2\pi)$ and the Hawking temperature is $T = \kappa/(2\pi)$, with κ the surface gravity.

What Paper 52 laid down: the 8π of T_{BH} is $4 \times 2\pi$, its 2π being a periodicity, distinct in root from the $8\pi = 2 \times 4\pi$ of the Einstein equations (a solid angle), and **sharing its root with Unruh's 2π** .

What this paper adds: a demonstration of the shared root, and a test of the decomposition. The corpus usually separates roots; this paper confirms one — confirmation is also a roll-call (Paper 100).

2. The 2π is a smoothness condition

Observation 2.1

The Euclidean Rindler metric is

$$ds^2 = \rho^2 d(a\eta)^2 + d\rho^2,$$

which near the origin is **a plane written in polar coordinates**, with $a\eta$ as the angle.

Verification 2.2 (the conical deficit): The deficit is 2π minus the period. This paper obtains

period of $a\eta$	deficit	
π	π	conical singularity
2π	0	smooth
4π	-2π	conical singularity

Only at exactly 2π does the cone disappear. (QED)

Observation 2.3

A period of 2π in $a\eta$ means a period of $2\pi/a$ in η , and since the period of imaginary time is the inverse temperature, $T = a/(2\pi)$.

So the address of this 2π is smoothness. It is not the total solid angle of a sphere (Papers 35, 45). An instance of the corpus's theme that one symbol arrives from different places.

3. The 4 lives in the surface gravity

Verification 3.1: For $f(r) = 1 - 2M/r$, the surface gravity at the horizon $r_h = 2M$ is

$$\kappa = \frac{f'(r_h)}{2} = \frac{1}{4M}$$

(computed here symbolically), and therefore

$$T = \frac{\kappa}{2\pi} = \frac{1}{8\pi M}.$$

(QED)

Observation 3.2

The 4 of $8\pi = 2\pi \times 4$ comes from the relation between surface gravity and mass, not from any periodicity. Paper 52's decomposition is confirmed concretely here.

4. Move it, and the 4 moves while the 2π does not

Verification 4.1 (charge it): For Reissner–Nordström, $\kappa = (r_+ - r_-)/(2r_+^2)$. This paper obtains

Q/M	κM	TM
0.00	0.250000	0.039789 (= $1/8\pi$)
0.50	0.248711	0.039584
0.90	0.211414	0.033648
0.99	0.108344	0.017243
1.00	0.000000	0.000000

(QED)

Verification 4.2 (spin it): For Kerr, $\kappa = (r_+ - M)/(r_+^2 + a^2)$.

a/M	κM	TM
0.000	0.250000	0.039789
0.500	0.232051	0.036932
0.900	0.151784	0.024157
0.998	0.029728	0.004731
1.000	0.000000	0.000000

(QED)

Observation 4.3 (this is the core)

The 8π is not universal. Charge it or spin it and the coefficient moves, vanishing at extremality. The division by 2π , meanwhile, never once changes.

So what is universal is $T = \kappa/(2\pi)$, not $T = 1/(8\pi M)$. Paper 52's decomposition has survived the test: on moving things, the moving part and the fixed part came apart.

Observation 4.4 (as a tool of the corpus)

Papers 117 and 132 said that **the separator is what has to be moved for a number to move. Here the separators are the charge and the angular momentum of the hole, and the same test pulls the 4 apart from the 2π .**

5. Near the horizon, the local temperature is Unruh's

Verification 5.1: By Tolman's law the local temperature is $T_{\text{loc}} = T_{\infty}/\sqrt{f}$, and the proper acceleration of a static observer is $a = M/(r^2\sqrt{f})$. This paper obtains

$r/2M - 1$	T_{loc}/a
10^{-2}	0.16235396
10^{-4}	0.15918678
10^{-6}	0.15915526
10^{-8}	0.15915495

agreeing with $1/(2\pi) = 0.15915494$ to eight digits. (QED)

Observation 5.2

The Hawking temperature measured locally near the horizon is exactly the Unruh temperature of the local acceleration. The two 2π are one and the same. Paper 52's claim is confirmed numerically as well.

6. They are nevertheless not the same phenomenon

Observation 6.1 (what differs)

	Unruh	Hawking
gravity	not needed	needed
a horizon of finite area	none	yes
evaporation, loss of mass	none	yes
where the 2π comes from	smoothness	smoothness (one root)

Observation 6.2 (one root is not one thing)

That the 2π share a root does not make the two the same phenomenon. The Unruh effect occurs in flat spacetime and brings with it no area law for entropy and no information problem. What Paper 17 asked for was an address, and two things can share an address while being different buildings.

Observation 6.3 (confirming a shared root is also a roll-call)

The corpus usually says that roots differ. But confirming that they agree belongs to the same discipline — both come from the single habit of not asserting what has not been checked. Saying that two things share a root also required moving them.

7. Conclusion

1. The 2π is a smoothness condition (Ver. 2.2). The Euclidean section avoids a cone **only at a period of exactly 2π** . It is not a solid angle.
2. The 4 lives in the surface gravity (Ver. 3.1). $\kappa = 1/(4M)$, hence $T = 1/(8\pi M)$. Paper 52's decomposition is confirmed concretely.
3. Move it and the 4 moves while the 2π does not (Ver. 4.1, 4.2). At $Q/M = 0.99$ the coefficient is **0.017243** and zero at extremality; Kerr the same. $T = \kappa/(2\pi)$ is universal and 8π is not.
4. Near the horizon the local temperature is Unruh's (Ver. 5.1): the ratio is **0.15915495**, eight digits of $1/(2\pi)$.
5. They are still not the same phenomenon (Obs. 6.1). Unruh has **no gravity, no finite-area horizon, no evaporation**. One root is not one thing.
6. Confirming a shared root is also a roll-call (Obs. 6.3), coming from the same habit of not asserting what has not been checked.

Closing: Move things, and only the 2π was left standing. Charge the hole or spin it, and the divisor does not change. What changed was the surface gravity, and the face called 8π was one that Schwarzschild happened to be wearing. There is only one way to tell what is universal — move it and see.

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