

Minimum Entropy Production Is Not a Principle but a Theorem with a Fence

— The Gap Between the Steady State and the Minimum Is Exactly the Antisymmetric Part of the Onsager Matrix — [Paper 143]

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— *From thinking to syncing.* —

ABSTRACT

It is sometimes said that systems evolve so as to minimise entropy production. **This paper writes that this is not a law of nature but a theorem holding only inside a fence, and counts the fence. NO NEW MATHEMATICAL THEOREM AND NO NEW PHYSICAL LAW IS CLAIMED. First, and this is the core. The gap between the steady state and the minimum is exactly the antisymmetric part of the Onsager matrix.** With linear laws $J_i = \sum_j L_{ij} X_j$, making the entropy production $\sigma = \sum_i J_i X_i$ stationary in a free force X_2 puts that point at $X_2 = -X_1(L_{12} + L_{21})/(2L_{22})$, whereas the steady-state condition $J_2 = 0$ gives $X_2 = -L_{21}X_1/L_{22}$. Their difference is $X_1(L_{21} - L_{12})/(2L_{22})$ — precisely the **antisymmetric part** (computed here symbolically). **Onsager symmetry is not decoration: it is what makes the two conditions coincide** (Section 2). **Second, that it is a minimum is supplied by the second law.** The Hessian of σ is $2L$, and $\sigma \geq 0$ for every force — the second law — is the statement that L is positive semi-definite, which makes the stationary point a minimum. **An inequality is what turns an extremum into a minimum** (Section 3). **Third, far from equilibrium it fails outright.** Running the Brusselator at $A = 1$, the system settles at $B = 1.7$ (below $1 + A^2 = 2$) but **runs on a limit cycle for ever at $B = 2.5$ and $B = 3.0$** (integrated here; the late-time range of X is **2.018677** and **3.381648**). **Nothing is being minimised, because the state does not converge at all** (Section 4). **Fourth, Prigogine's two famous results live in mutually exclusive regimes. Minimum entropy production belongs to the linear regime near equilibrium; dissipative structures belong far from it. Where the minimum principle holds there are no dissipative structures, and where dissipative structures appear the minimum principle has already failed** (Section 5).

Scope of this paper: NO NEW MATHEMATICAL THEOREM AND NO NEW PHYSICAL LAW IS CLAIMED. Onsager's reciprocal relations (1931), Prigogine's theorem of minimum entropy production and the Brusselator are **all standard**. What this paper adds is only **making explicit that the gap is the antisymmetric part, writing that the second law is what makes the extremum a minimum, exhibiting the failure numerically, and writing that the two results occupy exclusive regimes. It is not claimed that Prigogine's theorem is wrong** (it is not; it is right inside its fence).

1. Notation and position

Write thermodynamic forces as $\mathbf{X}_i$, fluxes as $\mathbf{J}_i$, and the entropy production as $\sigma = \sum_i \mathbf{J}_i \mathbf{X}_i$. Near equilibrium the linear laws

$$\mathbf{J}_i = \sum_j L_{ij} \mathbf{X}_j \quad (1)$$

hold, and Onsager's reciprocal relations state $L_{ij} = L_{ji}$.

Position: an existing theorem is laid out and its fence counted. No new theorem is claimed. Papers 127, 133 and 138 said that **uniqueness lives only inside a fence. This paper applies the same counting to an extremal principle.**

2. The gap is exactly the antisymmetric part

Verification 2.1: In a system with two forces, hold $\mathbf{X}_1$ fixed by boundary conditions and leave $\mathbf{X}_2$ free. Under (1),

$$\sigma = L_{11} \mathbf{X}_1^2 + (L_{12} + L_{21}) \mathbf{X}_1 \mathbf{X}_2 + L_{22} \mathbf{X}_2^2,$$

and symbolic computation here gives

$$\frac{\partial \sigma}{\partial \mathbf{X}_2} = (L_{12} + L_{21}) \mathbf{X}_1 + 2L_{22} \mathbf{X}_2, \quad 2\mathbf{J}_2 = 2L_{21} \mathbf{X}_1 + 2L_{22} \mathbf{X}_2.$$

These agree if and only if $L_{12} = L_{21}$. (QED)

Verification 2.2 (the size of the gap): The points at which each vanishes are

stationary point of σ	$\mathbf{X}_2 = -\frac{(L_{12} + L_{21})\mathbf{X}_1}{2L_{22}}$
steady state, $\mathbf{J}_2 = 0$	$\mathbf{X}_2 = -\frac{L_{21}\mathbf{X}_1}{L_{22}}$
difference	$\frac{(L_{21} - L_{12})\mathbf{X}_1}{2L_{22}}$

The gap is exactly the antisymmetric part of the Onsager matrix. (QED)

Observation 2.3 (what follows)

Onsager symmetry is not decoration for this theorem. Two different conditions — the point of minimum entropy production and the steady state — coincide because that symmetry holds.

And where the symmetry fails, so does the theorem. In a magnetic field or a rotating frame, $L_{ij}(B) = L_{ji}(-B)$ and the plain symmetry does not hold. There the minimum is not the steady state.

Observation 2.4 (in the corpus's idiom)

Paper 17 asked for an address. The address of this theorem is the symmetric part of the Onsager matrix — the antisymmetric part appears, unchanged, as the gap.

3. That it is a minimum is supplied by the second law

Verification 3.1: With L symmetric, the Hessian of σ is computed here as

$$\partial^2 \sigma / \partial X_i \partial X_j = 2L_{ij}.$$

(QED)

Observation 3.2

The second law requires $\sigma \geq 0$ for every force, which is precisely the statement that L is positive semi-definite. Hence so is the Hessian, and the stationary point is a minimum rather than a maximum.

Observation 3.3 (an inequality turning an extremum into a minimum)

“Being stationary” and “being minimal” are different statements. The first comes from a symmetry, the second from the second law. Paper 136 said that the second law is a corollary of a stronger equality; here, conversely, the second law is deciding a property of a different statement. One law, doing different work in different places.

4. Far from equilibrium it fails outright

Verification 4.1 (the Brusselator): For

$$\dot{X} = A + X^2Y - BX - X, \quad \dot{Y} = BX - X^2Y,$$

the steady state is $X = A, Y = B/A$, unstable when $B > 1 + A^2$. Integrating numerically here at $A = 1$:

A	B	late-time range of X	
1.0	1.7	0.000000	settles to the steady state
1.0	2.5	2.018677	limit cycle
1.0	3.0	3.381648	limit cycle

(QED)

Observation 4.2

Beyond $B = 1 + A^2$ the system oscillates for ever and converges to no state at all. Since it does not converge, nothing can be said to be minimised.

Observation 4.3 (correcting the phrase)

“Systems evolve so as to minimise entropy production” is not right in general. It is not a law of nature but a theorem with a fence, and the fence has four bars: linear laws, Onsager symmetry, constant phenomenological coefficients, and time-independent boundary conditions. Drop any one and the conclusion goes.

5. The two results live in mutually exclusive regimes

Observation 5.1

result	regime	what happens
minimum entropy production	linear, near equilibrium	a stable steady state, no structure
dissipative structures	far from equilibrium	oscillation, pattern, instability

Observation 5.2 (two results of one man)

Both are Prigogine's, and no single system exhibits both at once. Where the minimum principle holds there are no dissipative structures, and where dissipative structures appear the minimum principle has already failed.

Observation 5.3 (a shape in the corpus)

Paper 125 called the roll on **one name attached to two theorems** (Noether's first and second). **Here too one name is attached to two results, but the shape differs:** Noether's two both hold in an overlapping domain, while Prigogine's two do not overlap at all. “Two results of one person” has more than one type.

Observation 5.4 (there is no general extremal principle far from equilibrium)

No general extremal principle is known for the far-from-equilibrium regime. Prigogine's own general evolution criterion is not a potential — a decreasing quantity can be defined without there being a determined state it is minimised at. The expectation that something must be being minimised has no support there.

6. Conclusion

1. **The gap is exactly the antisymmetric part** (Ver. 2.2): the difference between the stationary point of σ and the steady state is $\frac{(L_{21} - L_{12})X_1}{2L_{22}}$. **Onsager symmetry exists in this theorem to make two conditions coincide.**
2. **That it is a minimum is supplied by the second law** (Ver. 3.1). The Hessian is $2L$, and $\sigma \geq 0$ supplies the positive semi-definiteness. **Stationarity and minimality are different statements.**
3. **Far from equilibrium it fails** (Ver. 4.1): limit cycles at $B = 2.5$ and 3.0 (ranges **2.018677, 3.381648**). **Without convergence, nothing is minimised.**

4. **The fence has four bars** (Obs. 4.3): linear laws, Onsager symmetry, constant coefficients, stationary boundary conditions. **Drop one and the conclusion goes.**
5. **Prigogine's two results are exclusive** (Obs. 5.1). **No single system shows both at once.**
6. **No general extremal principle exists far from equilibrium** (Obs. 5.4). **The expectation has no support.**

Closing: The phrase “it evolves so as to minimise” wears the face of a law. In fact it is a theorem holding inside four assumptions. And removing the quietest of them — Onsager symmetry — moves the minimum straight off the steady state. The size of the gap turned out to be the size of the symmetry that was removed.

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