

The Exponent Is the Signature of What Is Conserved

— The $2/5$ of the Sedov–Taylor Solution, an Exponent That Moves with the Dimension, and the γ That Dimensional Analysis Cannot See — [Paper 144]

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— *From thinking to syncing.* —

ABSTRACT

A shock driven by energy released into a uniform medium expands with its radius growing as the two-fifths power of time. It is the law used daily on supernova remnants. **This paper writes where that exponent comes from, and where it does not. NO NEW MATHEMATICAL THEOREM AND NO NEW PHYSICAL LAW IS CLAIMED.** First, the exponent moves with the dimension. Solving the dimensional equations for $R = E^a t^b \rho^c$ in d dimensions gives $R \propto (Et^2/\rho)^{1/(d+2)}$, that is $R \propto t^{2/(d+2)}$ (computed here symbolically): $2/3$ at $d = 1$, $1/2$ at 2 , $2/5$ at 3 , $1/3$ at 4 . The $2/5$ is not a constant of nature but the solution of a linear system in the exponents (Papers 28, 117) (Section 2). Second, and this is the core. The exponent is the signature of what is conserved. Conserved energy gives $t^{2/(d+2)}$ and conserved momentum gives $t^{1/(d+1)}$ — in three dimensions, $2/5$ against $1/4$. Computing the local logarithmic slope in closed form, this paper finds convergence to 0.40000000 and 0.25000000 by $t = 10^8$. When radiation carries the heat away, the shock leaves the $2/5$ law and joins the momentum-conserving one. The exponent is not a property of the explosion but a signature of the conservation law (Section 3). Third, dimensional analysis cannot see γ . There are four variables, R, E, t, ρ , and three dimensions, M, L, T , so there is exactly one dimensionless group and hence **exactly one undetermined constant**. And the adiabatic index γ is already dimensionless, so dimensional analysis never sees it at all. The relation reads $R(\rho/(Et^2))^{1/(d+2)} = C(\gamma)$, and $C(\gamma)$ comes only from solving the self-similar equations (Section 4). Fourth, so there are again three layers: the form (dimensional analysis, forced), $C(\gamma)$ (the self-similar solution, computed), and γ (the medium, an input). The same shape as Papers 140 and 141 (Section 5). Fifth, there is also self-similarity whose exponent does not come from dimensional analysis. For problems such as an imploding shock — self-similarity of the second kind — the exponent is fixed instead by the condition that the solution pass smoothly through a singular point, an eigenvalue problem. One word, “self-similar”, covers two different origins of the exponent (Paper 117) (Section 6).

Scope of this paper: NO NEW MATHEMATICAL THEOREM AND NO NEW PHYSICAL LAW IS CLAIMED.

Taylor (1950), Sedov (1946), von Neumann, and Barenblatt's distinction between the first and second kinds are **all standard**. What this paper adds is only **writing the exponent as the signature of a conservation law, deriving from the Buckingham count that γ is invisible, confirming the convergence of the exponent by local logarithmic slopes, and setting the two kinds side by side as distinct roots. The value of $C(\gamma)$ is not computed here — numbers that have not been computed are not written** (Paper 101).

1. Notation and position

Let an energy E be released at a point in a uniform medium of density ρ , and write the shock radius as $R(t)$, the background pressure being negligible.

Position: existing results are laid out and the origin of the exponent written down. No new theorem is claimed. Paper 28 said that **an exponent is not a dimension**, and Paper 117 that **scale invariance has four roots**. **This paper turns both on the Sedov–Taylor solution.**

The solution is treated here in the setting where it is used today, the expansion of supernova remnants.

2. The exponent moves with the dimension

Verification 2.1: Writing $R = E^a t^b \rho^c$ and solving the dimensional equations in d dimensions with $[E] = ML^2T^{-2}$ and $[\rho] = ML^{-d}$, symbolic computation here gives

$$a = \frac{1}{d+2}, \quad b = \frac{2}{d+2}, \quad c = -\frac{1}{d+2},$$

that is

$$R \propto \left(\frac{Et^2}{\rho} \right)^{1/(d+2)} \propto t^{2/(d+2)}.$$

(QED)

d	exponent of t
1	$2/3$
2	$1/2$
3	$2/5$
4	$1/3$

Observation 2.2

The $2/5$ is not a constant of nature. It is the solution of a linear system in the exponents, and it moves when the dimension changes. Paper 28 warned against mistaking an exponent for a dimension; here the exponent is a function of the dimension, and it is still not a dimension.

3. The exponent is the signature of what is conserved

Observation 3.1 (two conservation laws)

Taking the swept-up mass to be of order ρR^d ,

$$\text{energy conserved : } \rho R^d \dot{R}^2 = \text{const}, \quad \text{momentum conserved : } \rho R^d \dot{R} = \text{const},$$

which give

$$R \propto t^{2/(d+2)}, \quad R \propto t^{1/(d+1)}.$$

d	energy-conserving	momentum-conserving
1	$t^{2/3}$	$t^{1/2}$
2	$t^{1/2}$	$t^{1/3}$
3	$t^{2/5}$	$t^{1/4}$

Verification 3.2: Starting from $R(1) = 1$, both integrate in closed form as $R^p = 1 + p(t - 1)$, with $p = (d + 2)/2$ for energy and $p = d + 1$ for momentum. The local logarithmic slope $d \ln R / d \ln t$ runs

t	energy-conserving	momentum-conserving
10	0.42553191	0.27027027
10^2	0.40241449	0.25188917
10^4	0.40002400	0.25001875
10^8	0.40000000	0.25000000

(QED)

Observation 3.3 (this is the core)

The exponent alone distinguishes the two laws. Knowing nothing of the details of the explosion, taking logarithms of radius and time and reading the slope tells which quantity is conserved.

And real remnants change over. Once radiation begins carrying the heat away, energy is no longer conserved and the shock leaves the 2/5 law for the momentum-conserving one. The exponent is a signature not of “the explosion” but of what is conserved now.

Observation 3.4 (in the corpus's idiom)

Paper 135 said that whether an error is a drift or an oscillation is told by the scaling, not the size. The same holds here: the size says nothing and the exponent says everything.

4. Dimensional analysis cannot see γ

Observation 4.1 (the Buckingham count)

The variables are R, E, t, ρ — four — and the independent dimensions are M, L, T — three. So there is $4 - 3 = 1$ dimensionless group, that is, exactly one undetermined constant.

Observation 4.2 (adding γ adds no freedom to see it)

The adiabatic index γ is **already dimensionless**. Adding it makes the count $5 - 3 = 2$, and the relation reads

$$R\left(\frac{\rho}{Et^2}\right)^{1/(d+2)} = C(\gamma).$$

Dimensional analysis therefore says nothing whatever about γ . A dimensionless quantity is invisible to it from the start.

Observation 4.3 (the other half of the famous story)

Reading a radius and a time off a single photograph and recovering the energy by dimensional analysis alone is a famous story. It is true, and it is half. The other half is the number $C(\gamma)$, which depends on the medium and comes only from solving the self-similar equations.

The value of $C(\gamma)$ is not computed in this paper. Numbers that have not been computed are not written (Paper 101).

5. Again, three layers

Observation 5.1

what	what fixes it	standing
the form $R \propto (Et^2/\rho)^{1/(d+2)}$	dimensional analysis	forced
the constant $C(\gamma)$	the self-similar solution	computed
γ	the medium	an input

Observation 5.2

The same three layers as Paper 140 (symmetry, dynamics, phase) and Paper 141 (dimension, Lane–Emden, composition). Three papers in a row have produced the shape: a forced form, a number that must be computed, and an input that must be supplied. The common lesson is that a dichotomy does not suffice.

6. There is self-similarity whose exponent does not come from dimension

Observation 6.1 (first and second kind)

The exponent of the Sedov–Taylor solution follows **from dimensional analysis alone**. This is **self-similarity of the first kind**.

But for problems such as a shock converging on a centre, the exponent does not follow from dimensional analysis. There it is fixed by the condition that the solution pass smoothly through a singular point — an eigenvalue problem. This is self-similarity of the second kind (Barenblatt).

	first kind	second kind
what fixes the exponent	dimensional analysis	an eigenvalue condition
the value	rational (from the dimensions)	generally irrational
example	Sedov–Taylor	an imploding shock

Observation 6.2 (one word, two roots)

The single word “self-similar” covers two different origins of the exponent. Paper 117 listed **four roots of scale invariance**, of which root B is the dimensional matrix. The first kind belongs to root B and the second kind does not. The separator is whether the exponent follows from dimensional analysis.

Observation 6.3 (rationality as a clue)

Solutions of a dimensional matrix are rational, being solutions of a linear system. So an observed exponent that is irrational did not come from dimensional analysis. The rationality of the exponent is itself a clue to its root.

7. Conclusion

1. The exponent moves with the dimension (Ver. 2.1): $R \propto t^{2/(d+2)}$, giving $2/3, 1/2, 2/5, 1/3$ at $d = 1, 2, 3, 4$. The $2/5$ is not a constant of nature.

2. **The exponent is the signature of a conservation law** (Ver. 3.2): $\frac{2}{5}$ for energy and $\frac{1}{4}$ for momentum, the local slopes reaching **0.40000000** and **0.25000000** by $t = 10^8$. **Radiative losses move the shock from one to the other.**
3. **Dimensional analysis cannot see γ** (Obs. 4.2), γ being **already dimensionless**, so the relation retains $C(\gamma)$. **The famous story is half of one.**
4. **There are three layers** (Obs. 5.1): form (forced), $C(\gamma)$ (computed), γ (supplied) — **the same as Papers 140 and 141, a third time.**
5. **In self-similarity of the second kind the exponent does not come from dimension** (Obs. 6.1), being fixed by an **eigenvalue condition and generally irrational.**
6. **Rationality is a clue to the root** (Obs. 6.3): **solutions of a dimensional matrix are always rational.**

Closing: Take logarithms of radius and time, read the slope, and what is conserved right now can be read off. The exponent is not the violence of the explosion nor the detail of the medium but the signature of a conservation law. And when an exponent stops being rational, that is the sign that it came from outside dimensional analysis.

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