

Pressure Pulls Too, and That Is Why There Is a Bound

— Buchdahl's Bound Needs No Equation of State: a Bound on a Ratio, Not on a Mass — [Paper 145]

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— *From thinking to syncing.* —

ABSTRACT

A static star cannot be made arbitrarily small. **This paper writes why that bound exists, what it demands and does not demand, and what exactly it bounds. NO NEW MATHEMATICAL THEOREM AND NO NEW PHYSICAL LAW IS CLAIMED. First, and this is the core. In general relativity, pressure pulls too.** The Tolman–Oppenheimer–Volkoff equation is Newtonian hydrostatic balance multiplied by **three factors — the energy of the pressure itself, pressure as a source, and the curvature of space — and all three exceed one whenever $P > 0$.** In Newtonian gravity pressure only holds a star up; in general relativity it also pulls. Adding support strengthens the gravity as well, so beyond some compactness no amount of pressure can win (Section 2). **Second, the location of the bound follows in closed form.** For the uniform-density interior solution the central pressure is $P_c/(\rho c^2) = (1 - \sqrt{1-u})/(3\sqrt{1-u} - 1)$ with $u = 2GM/(Rc^2)$, and the denominator vanishes exactly at $u = 8/9$ (computed here symbolically). That is $R > \frac{9}{8}r_s$, Buchdahl's bound. At $u = 0.888$ the ratio is **166.7**, and it diverges as $u \rightarrow 8/9$ (Section 3). **Third, this bound needs no equation of state.** It needs only that the star be static, spherically symmetric, of isotropic pressure, with density non-increasing outward and non-negative. What the star is made of is not asked (Section 4). **Fourth, what is bounded is a ratio, not a mass.** Scaling M and R by the same factor leaves u unchanged. “Neutron stars have a maximum mass” is not Buchdahl's theorem — that needs an equation of state, and the equation of state at those densities is not known (Section 5). **Fifth, the corpus now holds three bounds demanding three different amounts of input: Chandrasekhar (Paper 141) demands an equation of state and bounds a mass; Buchdahl demands almost nothing and bounds a ratio; the horizon demands no matter at all and bounds a length** (Section 6).

Scope of this paper: NO NEW MATHEMATICAL THEOREM AND NO NEW PHYSICAL LAW IS CLAIMED.

Tolman (1939), Oppenheimer–Volkoff (1939), Buchdahl (1959) and the interior Schwarzschild solution are **all standard**.

What this paper adds is only **making explicit that all three TOV factors exceed one, locating the bound as the divergence of the uniform-density solution, writing that what is bounded is a ratio rather than a mass, and setting three bounds side by side by the input they demand.** Note the direction of $R > \frac{9}{8}r_s$ — the radius is the larger.

1. Notation and position

Write the Schwarzschild radius as $r_s = 2GM/c^2$ and the compactness as $u = r_s/R = 2GM/(Rc^2)$; $u \rightarrow 1$ is the horizon.

Position: existing theorems are laid out with their demands and their conclusions. No new theorem is claimed. Paper 126 said that **an existence theorem delivers existence and nothing more**, and Paper 141 wrote how **a limit is born from a coincidence of exponents**. This paper places a third bound beside them, sorted by the input it demands.

2. In general relativity, pressure pulls too

Observation 2.1 (three factors)

Newtonian hydrostatic balance is $dP/dr = -Gm\rho/r^2$. Its general-relativistic counterpart, the TOV equation, is

$$\frac{dP}{dr} = -\frac{Gm\rho}{r^2} \underbrace{\left(1 + \frac{P}{\rho c^2}\right)}_A \underbrace{\left(1 + \frac{4\pi r^3 P}{mc^2}\right)}_B \underbrace{\left(1 - \frac{2Gm}{rc^2}\right)^{-1}}_C.$$

Observation 2.2 (all three exceed one)

While $P > 0$ and $2Gm < rc^2$, $A > 1$, $B > 1$ and $C > 1$.

	what it adds	effect
<i>A</i>	the pressure carries energy	more inertia
<i>B</i>	pressure is a source of gravity	more pull
<i>C</i>	space is curved	distances are longer

Observation 2.3 (this is the core)

In Newtonian gravity pressure appears only on the supporting side. In general relativity it supports and it also pulls.

So the move “if it is about to collapse, raise the pressure” stops working partway. Raising the pressure raises *A* and *B*, strengthening the gravity too. Somewhere the added gravity beats the added support. That is why a bound exists.

Observation 2.4 (against Paper 141)

In Paper 141 the bound arose because **two forces carried the same power of the radius. Here it arises because the supporting side also joins the pulling side. One word, “limit”, naming two different mechanisms** (Paper 17).

3. Where the bound sits

Verification 3.1 (the uniform-density interior): In the interior Schwarzschild solution the central pressure is

$$\frac{P_c}{\rho c^2} = \frac{1 - \sqrt{1 - u}}{3\sqrt{1 - u} - 1}, \qquad u = \frac{2GM}{Rc^2},$$

and solving for the vanishing of the denominator gives ***u* = 8/9** (computed here symbolically). (QED)

Verification 3.2 (the divergence)

<i>u</i> = <i>r_s</i> / <i>R</i>	<i>R</i> / <i>r_s</i>	<i>P_c</i> /(<i>ρc</i> ²)
0.100	10.000	0.0278
0.500	2.000	0.2612
0.800	1.250	1.6180
0.880	1.136	16.6603
0.888	1.126	166.666
8/9[−]	1.125	divergent

(QED)

Observation 3.3 (note the direction)

The bound is $R > \frac{9}{8}r_s$ — the radius is the larger. That is $u < 8/9$: a star can exist only outside nine-eighths of its horizon.

Observation 3.4 (the standing of the 9/8)

The 9/8 is not a geometric quantity. It follows from the condition $\sqrt{1-u} = 1/3$. That 1/3 comes from the structure of the interior solution, from neither a sphere nor a solid angle (the discipline of Papers 35, 45).

4. This bound needs no equation of state

Observation 4.1 (what it demands)

Buchdahl's theorem demands only

that the star be static

that it be spherically symmetric

that the pressure be isotropic

that the density not increase outward

that the density be non-negative

No equation of state is required.

Observation 4.2 (this is unusual)

A bound on a geometric ratio is obtained without asking what the star is made of. The Chandrasekhar bound of Paper 141 required a specific equation of state, that of an ultra-relativistic degenerate gas. Here none is needed.

Observation 4.3 (but a light demand buys a light conclusion)

A theorem that demands little says little (Paper 126). Buchdahl speaks only of a ratio and says nothing whatever about the mass (Section 5).

5. What is bounded is a ratio, not a mass

Observation 5.1

Scaling M and R by the same factor k leaves $u = 2GM/(Rc^2)$ unchanged. Buchdahl's condition is therefore satisfied in the same way by stars of any size.

Observation 5.2 (correcting the phrase)

“Neutron stars have a maximum mass” is not Buchdahl's theorem. Naming a maximum mass requires an equation of state, and the equation of state at those densities is not known. What Buchdahl gives is a compactness that no equation of state can exceed.

Observation 5.3 (write what is being bounded)

Paper 17 asked for an address. Saying that there is a limit requires saying a limit on what — a mass, a radius, or a ratio. The three are different statements.

6. Three bounds, three inputs

Observation 6.1

bound	what it demands	what it bounds
Chandrasekhar (Paper 141)	an equation of state	a mass (with μ_e)
Buchdahl (here)	no equation of state	a ratio ($R/r_s > 9/8$)
the horizon	no matter at all	a length (r_s)

Observation 6.2 (the less demanded, the less said)

Chandrasekhar names a definite mass, at the price of demanding microphysics. Buchdahl demands almost nothing, at the price of naming only a ratio. The horizon demands nothing and names only a length.

One word, “limit”, naming three different statements. Demand and conclusion are in balance — and without seeing that balance one cannot tell which bound supports which claim.

7. Conclusion

1. In general relativity pressure pulls too (Obs. 2.2). TOV carries three factors, all exceeding one when $P > 0$. Adding support strengthens gravity — that is why a bound exists.
2. The bound sits at $u = 8/9$ (Ver. 3.1). The central pressure of the uniform-density solution diverges where $3\sqrt{1-u} - 1$ vanishes, at $u = 8/9$, that is $R > \frac{9}{8}r_s$ (note the direction).
3. The $9/8$ is not geometric (Obs. 3.4), following from $\sqrt{1-u} = 1/3$.
4. No equation of state is required (Obs. 4.1): only static, spherical, isotropic, non-increasing and non-negative density.
5. What is bounded is a ratio (Obs. 5.1); scaling M and R together leaves u fixed. A maximum mass is not Buchdahl's.
6. The three bounds balance demand against conclusion (Obs. 6.1). The less input, the less said (Paper 126).

Closing: In the Newtonian world, if a thing was about to collapse one raised the pressure. In general relativity that move tightens the noose, because what was added to support also joins the pull. A bound is the point at which the support becomes its own enemy.

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