

The Same H^2 , and with Different Coefficients Nothing Is Quantised

— A Census of the Obstructions from Local to Global: Stokes, Gauss, Dirac, Poincaré–Hopf — [Paper 147]

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— *From thinking to syncing.* —

ABSTRACT

The corpus has met the shape “locally trivial, globally not” many times (Papers 92, 70, 128, 146). **This paper calls the roll on those obstructions and writes their addresses and types. NO NEW MATHEMATICAL THEOREM AND NO NEW PHYSICAL LAW IS CLAIMED.** First, the obstruction is a group, and its dimension is not any of the corpus's numbers. For the circle, the sphere, the torus and punctured three-space alike, the top cohomology is **one-dimensional**. The corpus's 4π is not that dimension — it is the value of a pairing, a period, not the size of a group (Section 2). Second, being one-dimensional is exactly why the area form and the Euler form are proportional. On a sphere of radius R the area form integrates to $4\pi R^2$ and $K dA$ to 4π , a ratio of R^2 (computed here symbolically). Inside one one-dimensional group, one depends on the metric and one does not. Paper 128's separation is visible here as a factor R^2 inside a single group (Section 3). Third, 2π and 4π appear as periods of generators. The generator of H^1 of the circle has period 2π , and that of H^2 of the unit sphere has period 4π (computed here). But this is a shared language, not a shared root — Paper 142's 2π is a smoothness condition and Paper 128's 4π is $\text{Vol}(S^2)$, and they come from different places (Section 4). Fourth, and this is the core. The same H^2 with different coefficients quantises nothing. $H^2(S^2; \mathbb{R}) = \mathbb{R}$ is a continuum and permits any flux; $H^2(S^2; \mathbb{Z}) = \mathbb{Z}$ is a lattice and permits only integer classes. De Rham cohomology, having real coefficients, quantises nothing at all (Section 5). Fifth, so electric charge is not quantised by Gauss's law. It is quantised only if a monopole exists, and then by the same integer read the other way round: $2eg/\hbar = n$ constrains the product, so one monopole quantises every electric charge (Section 6).

Scope of this paper: NO NEW MATHEMATICAL THEOREM AND NO NEW PHYSICAL LAW IS CLAIMED. De Rham's theorem, the agreement of Čech and de Rham cohomology, the exponential sequence and Dirac's quantisation are all standard. What this paper adds is only setting four obstructions side by side with their addresses and types, making explicit that 4π is a period and not a dimension, exhibiting as a factor R^2 that a metric-dependent and a topological form share one one-dimensional group, and writing that the separator for quantisation is the coefficients. It is not claimed that monopoles exist (as in Paper 146).

1. Notation and position

Write de Rham cohomology with real coefficients as $H^k(M;\mathbb{R})$ and singular cohomology with integer coefficients as $H^k(M;\mathbb{Z})$.

Position: existing theorems are laid side by side with addresses and types. No new theorem is claimed. Paper 146 **named the failure to glue in its smallest example; this paper extends that roll-call to the corpus's other sites.**

2. The obstruction is a group, and its dimension is not the corpus's number

Observation 2.1

space	de Rham cohomology	top dimension	the corpus's number
S^1	$H^0 = \mathbb{R}, H^1 = \mathbb{R}$	1	2π
S^2	$H^0 = \mathbb{R}, H^1 = 0, H^2 = \mathbb{R}$	1	4π
T^2	$H^0 = \mathbb{R}, H^1 = \mathbb{R}^2, H^2 = \mathbb{R}$	1	the area
$\mathbb{R}^3 \setminus \{0\}$	$H^0 = \mathbb{R}, H^2 = \mathbb{R}$	1	4π

Observation 2.2 (a dimension and a period are different quantities)

The dimension is 1 in every case. The corpus's 4π is not that dimension but the value obtained by integrating a generator, that is, a period.

“The obstruction is one-dimensional” and “the number is 4π ” are two different statements about the same H^2 (Paper 17): one is the size of a group and the other the value of a normalisation.

3. One-dimensional, so the two forms are proportional

Verification 3.1: On a sphere of radius R , symbolic computation here gives

form	integral	standing
the area form $R^2 \sin \theta$	$4\pi R^2$	metric-dependent
$K dA = \sin \theta$	4π	topological
ratio	R^2	

(QED)

Observation 3.2 (this is the point)

Since $H^2(S^2)$ is one-dimensional, these two 2-forms are elements of one group and constant multiples of each other. Belonging to the same family of cohomology classes says nothing about which of them is forced.

Paper 128 separated $\text{Vol}(S^2)$ from $2\pi\chi(S^2)$. Here that separation appears as a factor R^2 inside a single group. “Living in the same group” is not “being the same thing”.

4. 2π and 4π are periods of generators

Verification 4.1: The computation here gives

group	generator	period
$H^1(S^1)$	$d\varphi$	2π
$H^2(S^2)$ (unit sphere)	$\sin \theta d\theta \wedge d\varphi$	4π

(QED)

Observation 4.2 (a shared language, not a shared root)

In this language 2π and 4π line up as periods of generators one degree apart. That does not make them share a root.

Paper 142's 2π is the smoothness condition that removes a conical singularity, and Paper 128's 4π is the volume of the space of directions. Being shelved together and coming from the same place are different things (Paper 100). The

shelf is a language; the provenance is a root.

5. The same H^2 with different coefficients quantises nothing

Observation 5.1

	coefficients	what is permitted
$H^2(S^2; \mathbb{R}) = \mathbb{R}$	real	a continuum — any flux
$H^2(S^2; \mathbb{Z}) = \mathbb{Z}$	integer	a lattice — integer classes only

Observation 5.2 (this is the core)

De Rham cohomology, having real coefficients, quantises nothing. Integrality comes from the integer class — equivalently, from requiring a $U(1)$ transition function to be single-valued (Paper 146).

So “it is quantised because there is a global obstruction” is not right. That an obstruction exists and that its value comes in steps are different statements. The separator is the coefficients.

6. Electric charge is not quantised by Gauss's law

Observation 6.1

Gauss's law $\oint \mathbf{E} \cdot d\mathbf{A} = q/\epsilon_0$ places no restriction whatever on q . The flux runs over a real continuum.

Observation 6.2 (then why is charge quantised)

Not by this argument. If even one monopole exists, $2eg/\hbar = n$ constrains the product, so fixing g quantises every e (Dirac). But no monopole has been found.

Observation 6.3 (the same integer, read the other way)

The integer read in Paper 146 as the quantisation of g is read here as the quantisation of e . It is one and the same integer, and the roles swap according to which quantity is taken as known (the same shape as Paper 16's roots of the equals sign: **one relation becomes a different statement depending on where one stands**).

7. The census

Observation 7.1

where	the obstruction's address	type	behaviour
Stokes (Paper 92)	$H^k = 0$	—	no obstruction (closed is exact)
Gauss's law	$H^2_{\text{dR}}(\mathbb{R}^3 \setminus \{0\}; \mathbb{R})$	real	moves continuously with the charge
Dirac (Paper 146)	$H^2(S^2; \mathbb{Z})$	integer	moves in steps
Poincaré–Hopf (Paper 128)	the Euler class in $H^2(M; \mathbb{Z})$	integer	is $\chi(M)$

Observation 7.2

One phrase, “locally trivial but globally not”, names four different obstructions. And what separates the second from the third is neither the space nor the dimension but the coefficients.

Observation 7.3 (what the sheaf does)

A sheaf keeps the books on local data and their agreement on overlaps. A common bookkeeping does not make the entries the same. What Grothendieck's viewpoint supplies is a common language, not a common root — and what the corpus has been guarding is the refusal to read an agreement of language as an agreement of root.

8. Conclusion

1. **The obstruction is a group and the dimension is 1 (Obs. 2.1). The corpus's 4π is not that dimension but the period of a generator.**
2. **Being one-dimensional, the two forms are proportional (Ver. 3.1): $4\pi R^2$ against 4π , a ratio of R^2 . Living in the same group is not being the same thing (Paper 128).**
3. **2π and 4π line up as periods (Ver. 4.1), which is a shared language and not a shared root (Paper 100).**
4. **The same H^2 with different coefficients quantises nothing (Obs. 5.1): real coefficients give a continuum, integer coefficients a lattice.**
5. **Electric charge is not quantised by Gauss's law (Obs. 6.1); a monopole would quantise it by the same integer read the other way (Paper 146).**
6. **Four obstructions line up (Obs. 7.1), and the second and third are separated by the coefficients.**

Closing: Being shelved together is not coming from the same place. Sheaves and cohomology give a common name to a shape the corpus has met again and again. But a common name with different coefficients yields different conclusions — moving continuously, or moving in steps. To write an address is to write not only the group but the coefficients.

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