

What Fixes the Denominator Is Not the Determinant but the Largest Invariant Factor

— What Pulls Back and What Pushes Forward: Functoriality and the Address of the Corpus's $|\det|$ — [Paper 149]

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— *From thinking to syncing.* —

ABSTRACT

This paper applies a brake to the corpus itself. Paper 39 wrote that the exponents of the Planck system are half-integers because the dimension-exponent matrix of $(\hbar, G, c)$ has $|\det| = 2$; Paper 03 wrote that the Kolmogorov scales come in quarters because that of (ν, ϵ) has $|\det| = 4$; and **both called this one and the same theorem. That theorem is a theorem of upper bounds, not of equalities. First, transporting a quantity along a map is not one operation.** A differential form **pulls back** along any smooth map, while a vector field cannot be transported at all — under $f(x) = x^2$ the point $y = 1$ receives **two answers**, $+2$ and -2 (Section 2). **Second, what separates a form from a density is orientation.** Under a reflection **the form changes sign and the measure does not**: the first carries **det** and the second $|\det|$ (Section 3). **Third, the address of the corpus's $|\det|$ is a volume in exponent space.** A dimension-exponent matrix is **a map of lattices** and $|\det|$ is its index, the volume of a fundamental domain (confirmed here by counting lattice points). **The absolute value is there because a volume has no orientation — the same root as the $|\det|$ of Section 3** (Section 4). **Fourth, and this is the core. $|\det|$ is an upper bound on the denominator, not the denominator. Counterexample: $\text{diag}(2, 2)$ has $|\det| = 4$, yet its inverse has only halves and no quarter appears anywhere. The denominator that actually occurs is the largest invariant factor d_n , and $d_n = |\det|$ exactly when the cokernel is cyclic** (Section 5). **Fifth, both of the corpus's instances turned out to be cyclic.** Paper 03's matrix has invariant factors $(1, 4)$ and Paper 39's has $(1, 1, 2)$ (recomputed here). **So both conclusions stand. What was not right was the reason, which was stated with one invariant too coarse** (Section 6).

Scope of this paper: NO NEW MATHEMATICAL THEOREM IS CLAIMED. The Smith normal form, the equality of a lattice index with a determinant, and the functoriality of pullback and pushforward are **all standard**. What this paper adds is **giving the corpus's $|\det|$ the address “a volume in exponent space”, writing that it shares a root with the $|\det|$ on a density, stopping “ $|\det|$ fixes the denominator” with a counterexample, and recomputing the corpus's two instances to confirm that their conclusions survive. The conclusions of Papers 03 and 39 are not withdrawn; only the way the reason is written is corrected.**

1. Notation and position

Write f^* for pullback and f_* for pushforward along a smooth map f , and $d_1 \mid d_2 \mid \cdots \mid d_n$ for the invariant factors of an integer matrix (the diagonal of its Smith normal form).

Position: Papers 146 and 147 treated the address of an obstruction and Paper 148 the distinct roots of “point”. **This paper treats orientation — and gives an address to the symbol $|\det|$ that the corpus has used in three papers, adding one restriction on how it may be used.**

2. What pulls back and what pushes forward

Verification 2.1 (a form pulls back): For $f(x) = x^2$ one has $f^*(dy) = 2x \, dx$, which is defined at every point. (QED)

Verification 2.2 (a vector field cannot be transported): Pushing ∂_x forward along the same f gives $f_*(\partial_x) = 2x \, \partial_y$, so

x	$y = f(x)$	$f_*(\partial_x)$
+1	1	$+2 \, \partial_y$
−1	1	$−2 \, \partial_y$

That is, **one point $y = 1$ receives two different vectors.** (QED)

Observation 2.3

Forms are contravariant and pull back along any map. Vector fields are covariant and need a diffeomorphism. This is not even a question of orientation but of injectivity.

Verification 2.4 (a measure pushes forward): Pushing the uniform measure on $[0, 1]$ forward by $f(x) = x^2$ gives density $1/(2\sqrt{y})$. With 4×10^6 samples,

y	empirical density	$1/(2\sqrt{y})$	rel. error
0.225	1.0527	1.0541	0.0014
0.375	0.8160	0.8165	0.0006
0.675	0.6081	0.6086	0.0008
0.975	0.5061	0.5064	0.0005

(QED) What the density picks up is $1/|f'|$ and not $1/f'$.

3. What separates a form from a density is orientation

Verification 3.1: The reflection $T(x, y) = (y, x)$ has $\det T = -1$.

what is transported	factor	sign
the area form $dx \wedge dy$	$\det T = -1$	flips
the area measure	$ \det T = 1$	does not

Verification 3.2: For B with rows $(2, 1)$ and $(1, 3)$ one has $\det B = 5$, and measuring the area of the image of the unit square on a 4000×4000 grid gives **5.000239**. (QED)

Observation 3.3

One symbol carries two transport rules, and orientation separates them. A measure is a positive quantity and has no orientation.

4. The address of the corpus's $|\det|$ is a volume in exponent space

Verification 4.1: Counting how many points of a 121×121 window lie in the sublattice spanned by Paper 03's dimension-exponent matrix (rows $(2, 2)$ and $(-1, -3)$) gives **3691 of 14641**, a ratio of **0.2521** including boundary effects, matching $1/|\det| = 0.25$. (QED)

Observation 4.2

The index of a sublattice is the volume of a fundamental domain, and a volume has no orientation. So the absolute value in the corpus's $|\det|$ shares its root with the $|\det|$ on a density in Section 3 — both say that a volume cannot be oriented. This is a rare occasion on which the corpus confirms a shared root rather than separating one (the shape of Paper 142).

Observation 4.3 (write the address to the end)

This volume is not a physical volume. It is a volume in the lattice of dimension exponents, the space made by the logarithms of units. Without writing that, the $|\det|$ starts to look like some physical quantity (Paper 17).

5. $|\det|$ is an upper bound on the denominator, not the denominator

Fact 5.1: For an integer matrix, $A^{-1} = \text{adj}(A)/\det A$ with $\text{adj}(A)$ integral, so the denominators of the entries of A^{-1} divide $|\det A|$. This is an upper bound and not an equality.

Counterexample 5.2: $A = \text{diag}(2, 2)$ has $|\det A| = 4$ but $A^{-1} = \text{diag}(\frac{1}{2}, \frac{1}{2})$, so no quarter appears anywhere. (QED)

Observation 5.3

Hence “ $|\det| = 4$, therefore fourth roots” does not hold as stated. The counterexample is not contrived — take an area (L^2) and a squared time (T^2) as the basis and the matrix is exactly $\text{diag}(2, 2)$. Building any quantity from them needs square roots at worst, and a fourth root never appears.

Theorem 5.4 (the denominator that occurs)

The least common multiple of the denominators of A^{-1} equals the largest invariant factor d_n of A . And $d_n = |\det A|$ holds exactly when the cokernel $\mathbb{Z}^n/A\mathbb{Z}^n$ is cyclic. (A standard consequence of the Smith normal form.)

6. Both of the corpus's instances were cyclic

Verification 6.1: Computing the invariant factors here,

matrix	$ \det $	invariant factors	d_n	denominator of A^{-1}
the counterexample $\text{diag}(2, 2)$	4	2, 2	2	2
Paper 03, (ν, ε)	4	1, 4	4	4
Paper 39, $(\hbar, G, c)$	2	1, 1, 2	2	2
Paper 38, (G, m, c)	1	1, 1, 1	1	1

Explicitly, Paper 03's inverse has rows $(\frac{3}{4}, \frac{1}{2})$ and $(-\frac{1}{4}, -\frac{1}{2})$, and Paper 39's has rows $(\frac{1}{2}, -\frac{1}{2}, \frac{1}{2})$, $(\frac{1}{2}, \frac{1}{2}, -\frac{3}{2})$ and $(\frac{1}{2}, \frac{1}{2}, -\frac{5}{2})$. (QED)

Observation 6.2 (the conclusions survive)

Paper 03's cokernel is $\mathbb{Z}/4$ and quarters really do occur. Paper 39's cokernel is $\mathbb{Z}/2$ and half-integers really do occur. Neither conclusion is withdrawn.

Observation 6.3 (what is corrected is the reason)

The form in which Paper 03 wrote it, “ $|\det| = 2$ gives square roots and $|\det| = 4$ gives fourth roots”, reads an upper bound as an equality. The correct statement is that a largest invariant factor of 2 gives square roots and of 4 gives fourth roots, $|\det|$ being merely a multiple of it. The two agreed in both instances because both cokernels were cyclic.

Observation 6.4 (a familiar shape)

A brake laid down by a first edition had not been applied to the first edition itself — the shape of Papers 56 to 60, here with Paper 17's “write the address” turned on the corpus's own determinant.

7. The census

Observation 7.1

object	direction	what it needs	factor carried
a function	pulls back	any map	—
a differential form	pulls back	any map	det
a vector field	neither	a diffeomorphism	—
a measure or density	pushes forward	any map	 det
cohomology	pulls back (contravariant)	any map	Paper 147
homology	pushes forward (covariant)	any map	the pairing

Observation 7.2

“Transporting a quantity along a map” is not one operation. The direction is not a matter of taste but is forced by what the object is.

Observation 7.3 (three papers in a row)

Paper 147's separator was the coefficients, Paper 148's the test object, and this paper's is orientation. All three have the shape: that a thing can be written in a common language does not make it share a root.

8. Conclusion

1. Forms pull back and vector fields cannot be transported (Ver. 2.1, 2.2): $y = 1$ receives both $+2$ and -2 .
2. Orientation separates a form from a density (Ver. 3.1): **det** against **|det|**.
3. The corpus's **|det|** is a volume in exponent space (Ver. 4.1), sharing the root of the density's **|det|**: a volume cannot be oriented.
4. **|det|** is an upper bound on the denominator, not the denominator (Ctrex. 5.2): **diag(2, 2)** has **|det| = 4** and yields only halves. What decides is the largest invariant factor (Thm. 5.4).
5. Both of the corpus's instances were cyclic (Ver. 6.1): $\mathbb{Z}/4$ for Paper 03 and $\mathbb{Z}/2$ for Paper 39. The conclusions survive and the reason is corrected.
6. The direction of transport is forced by the object (Obs. 7.1).

Closing: The corpus wrote three times that $|\det|$ is where a root of a given order comes from. Every one of the numbers was right. But the reason that produced the right numbers was one invariant too coarse. Reading an upper bound as an equality cannot be caught while the two happen to agree — to catch it one has to build, oneself, an example in which they do not.

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