

A Convention Is a Number, Not a Function of the Configuration

— 3rd Edition: Where the Bohr–Yamagishi Equation Holds — [Paper 15]

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— *From thinking to syncing.* —

ABSTRACT

This paper applied two "patches", $m = 4\pi$ and $\epsilon_0 = r$, to the balance equation of the Bohr model and obtained the form $16\pi^2 v^2 r^2 = e^2$, that is $(4\pi v)r = e$. The 3rd edition shows that **these two patches are not operations of the same kind, and fixes the range in which the equation holds. First, a convention is a number, not a function of the configuration. $m = 4\pi$ fixes the unit of mass once, and is a legitimate choice of units. But $\epsilon_0 = r$ assigns a different value at every radius. A system of units cannot depend on where the electron is. Hence $\epsilon_0 = r$ is not a choice of units** — among the three roots set out in Paper 38 (**forced / choice of units / choice of configuration**) it belongs not to the second but to the third. **Second, read as a change in the force law, $\epsilon_0 = r$ brings on the very collapse it was meant to prevent.** Substituting $\epsilon_0 = r$ into the Coulomb force $e^2/(4\pi\epsilon_0 r^2)$ gives $e^2/(4\pi r^3)$ — **an inverse-cube, not an inverse-square, law.** The effective potential of an inverse-cube force is $U_{\text{eff}} = (L^2/m - K)/2r^2$, which **admits no stable circular orbit** (the particle escapes, or falls to the centre, or every radius is neutrally in equilibrium). **Bertrand's theorem permits closed orbits only for the inverse square and for Hooke's law; the inverse cube lies outside it. Third, the extension to general principal quantum number cannot be derived. No n appears anywhere in the patched balance $16\pi^2 v^2 r^2 = e^2$, whereas the quantisation condition gives $4\pi v_n r_n = n\hbar$. For the two to hold together one would need $e = n\hbar$ for every n , a contradiction. Theorem 3.1 of the second edition ($16\pi^2 v_n^2 r_n^2 = e^2/n^2$, together with the claim that it "perfectly explains the Balmer series") is withdrawn. Fourth, something does survive. Read as numerical substitutions at one radius, $m = 4\pi$ and $\epsilon_0 = r$ make $(4\pi v)r = e$ a valid restatement at that radius. The equation is also dimensionally consistent within the system of units it implicitly defines — although in that system charge has the dimension of area per unit time. Fifth, why atoms do not collapse is already explained by quantum mechanics. The probability density of a stationary state is time-independent, so there is no oscillating dipole moment and it cannot radiate in principle (Schrödinger 1926). This paper does not replace that explanation. Scope: a structural account within established physics (Layer 2). This paper claims no explanation of atomic stability, none of spectra, and no new physical law.**

Positioning of this paper (discipline): This paper offers a **structural interpretation** of established physical laws. It claims neither the discovery of new physical laws nor the proof of open problems. The relations shown are transformations of known expressions; the aim is to propose a structural reading of them. No claim contradicting established theory is made. **The appearance of the same number in several contexts is not treated as evidence of a common origin** (Papers 48, 86, 100, 115). $m = 4\pi$ and the like are conventional choices within a system of units, not new facts about nature (Papers 35, 38, 95).

1. Introduction: what this equation restated

In the Rutherford model, an orbiting electron must radiate continuously and fall into the nucleus. **Computed here**, the classical collapse time from Larmor's formula is

$$t = \frac{c^3 m_e^2 a_0^3}{4k^2 e^4} = 1.556 \times 10^{-11} \text{ s}$$

(with $k = 1/4\pi\epsilon_0$). That was the question the first edition raised.

The self-consistency of the plain Bohr model is confirmed too (computed here): $a_0 = 5.291772 \times 10^{-11} \text{ m}$, $v_1 = 2.187691 \times 10^6 \text{ m/s} = 0.007297 c$, the ratio of mv^2/r to $e^2/(4\pi\epsilon_0 r^2)$ is 1.000000000000, and $mv_1 a_0/\hbar = 1.000000000000$. The fine structure constant comes out as $1/\alpha = 137.035999$.

This paper applied two patches, $m = 4\pi$ and $\epsilon_0 = r$, to that balance and obtained $16\pi^2 v^2 r^2 = e^2$, that is $(4\pi v) r = e$.

The algebra itself is correct.

What the 3rd edition asks is whether the two patches have the same standing.

2. A convention is a number, not a function of the configuration

Observation 2.1 (The two patches are not operations of the same kind)

$m = 4\pi$: this fixes the unit of mass once. Thereafter, for every electron and at every radius, m is the same 4π . This is a legitimate choice of units, of the same standing as $G = 4\pi$ and $\hbar = 1/4\pi$ in Papers 35, 38 and 39.

$\epsilon_0 = r$: this assigns a different value at every radius. The permittivity of the vacuum would differ by a factor of ten between an electron at $r = 10^{-11}$ m and one at $r = 10^{-10}$ m.

A system of units cannot depend on where the electron is. Hence $\epsilon_0 = r$ is not a choice of units.

Corollary 2.2 (Placement among the three roots of Paper 38)

The 3rd edition of Paper 38 sorted the numbers appearing in this series into **those that are forced** (the 4π of Gauss's flux, and so on), **choices of units**, and **choices of configuration**.

$m = 4\pi$ belongs to the second root; $\epsilon_0 = r$ to the third. The first and second editions grouped the two under the single word "Yamagishi patch" — grouping what must not be grouped.

Established Fact 2.3 (The budget of units)

Including charge, the base dimensions are M, L, T, Q — four — so at most four values may be freely assigned. This series has already spent three: $m = 4\pi$, $G = 4\pi$ and $\hbar = 1/4\pi$ (Papers 38, 39).

ϵ_0 could occupy the fourth slot — but only so long as it is a number. $\epsilon_0 = r$ does not meet that condition.

3. Read as a force law, $\epsilon_0 = r$ brings on the collapse it was meant to prevent

Reading ϵ_0 literally as a function of r turns the Coulomb force into

$$F = \frac{e^2}{4\pi\epsilon_0 r^2} \longrightarrow \frac{e^2}{4\pi r^3},$$

an inverse cube, not an inverse square.

Established Fact 3.1 (An inverse-cube force admits no stable circular orbit)

For an attractive $F = -K/r^3$ the effective potential is

$$U_{\text{eff}}(r) = \frac{L^2}{2mr^2} - \frac{K}{2r^2} = \frac{L^2/m - K}{2r^2},$$

which **has no minimum**, because the centrifugal term and the force term both go as r^{-2} . Only three cases exist (verified here):

condition	U_{eff}	consequence
$L^2/m > K$	$+0.50/r^2$	repulsive: no bound circular orbit (escape)
$L^2/m = K$	0 (identically)	every radius neutrally in equilibrium
$L^2/m < K$	$-0.25/r^2$	$\rightarrow -\infty$ as $r \rightarrow 0$: fall to the centre

This is the classic case Landau and Lifshitz treat as "**fall to the centre**".

Corollary 3.2 (Outside Bertrand's theorem)

Bertrand's theorem (1873) permits closed orbits only for the inverse-square force and for the harmonic oscillator (Hooke's law). The inverse cube lies outside it. The 3rd edition of Paper 28 cited Bertrand's theorem as a signature of what is special about the Kepler problem; here the same theorem works on the side of prohibition.

$\varepsilon_0 = r$ cannot be read as a change in the force law.

So read, the very collapse the first edition set out to prevent occurs — and already at the classical level, before quantum mechanics is reached.

4. The extension to general principal quantum number cannot be derived

Observation 4.1 (No n appears in the patched equation)

Substituting $m = 4\pi$ gives $4\pi v^2/r = e^2/(4\pi\epsilon_0 r^2)$, that is $4\pi v^2 = e^2/(4\pi\epsilon_0 r)$; substituting $\epsilon_0 = r$ then gives

$$16\pi^2 v^2 r^2 = e^2.$$

The principal quantum number n appears nowhere on the right. The balance equation does not know about n .

Corollary 4.2 (Incompatible with the quantisation condition)

Substituting $m = 4\pi$ into $mvr = n\hbar$ gives $4\pi v_n r_n = n\hbar$, while the balance gives $4\pi v_n r_n = e$. For both to hold one needs $e = n\hbar$ for every n .

Verified here: $n\hbar$ is 1.05×10^{-34} , 2.11×10^{-34} , 3.16×10^{-34} (J·s) for $n = 1, 2, 3$, whereas $e = 1.60 \times 10^{-19}$ (C) — neither the values nor the units agree.

Theorem 3.1 of the second edition — $16\pi^2 v_n^2 r_n^2 = e^2/n^2$ — therefore cannot be derived. The n in e^2/n^2 was written by analogy with the quantisation condition, not obtained from the balance. **The 3rd edition withdraws it.**

Observation 4.3 ("Perfectly explains the Balmer series" is withdrawn with it)

Following Theorem 3.1 the second edition stated that "the discrete spectral structure of the Balmer series and the like is perfectly explained as standing-wave modes inside the toroidal cocoon". That claim is withdrawn too.

For two reasons. (i) The theorem it presupposed cannot be derived. (ii) Even had the n -dependence been right, it would only have rewritten the quantisation condition $mvr = n\hbar$; the origin of the discreteness would still be Bohr's postulate — a change of notation, not an added explanation.

5. What survives — a restatement at one radius, and its system of units

Theorem 5.1 (The range in which it holds)

Read as **numerical substitutions at that radius**, $m = 4\pi$ and $\epsilon_0 = r$ put the Bohr balance condition into the form

$$(4\pi v)r = e.$$

This is a restatement valid at one radius; it can neither be differentiated with respect to r nor extended to other radii.

Established Fact 5.2 (The system of units it implies — charge becomes area per unit time)

We have $[\epsilon_0] = C^2 T^2 M^{-1} L^{-3}$. Setting $\epsilon_0 = r$, i.e. $[\epsilon_0] = L$, gives

$$C^2 T^2 M^{-1} L^{-3} = L \implies C^2 = M L^4 T^{-2} \implies C = M^{1/2} L^2 T^{-1},$$

and with $m = 4\pi$ (dimensionless), $M = 1$, so $C = L^2 T^{-1}$: area per unit time.

Hence $(4\pi v)r = e$ is dimensionally consistent as $(L/T)(L) = L^2/T$ — the equation is correct within the system of units it defines. But in that system, charge is an area per unit time.

This is a worked instance of Paper 95 (Poincaré's conventionalism). One is free to adopt a convention, but not free to choose what the convention drags along with it. Here what is dragged along is the dimension of charge itself.

6. Why atoms do not collapse is already explained by quantum mechanics

Established Fact 6.1 (A stationary state cannot radiate, in principle)

In quantum mechanics the probability density of a stationary state $\psi_n(\mathbf{r})e^{-iE_n t/\hbar}$ is

$$|\psi_n(\mathbf{r}, t)|^2 = |\psi_n(\mathbf{r})|^2,$$

which is **independent of time**. The charge density therefore does not vary in time, and **no oscillating dipole moment exists**. Since radiation arises from time-varying currents and charge distributions, **a stationary state cannot radiate in principle** (Schrödinger 1926).

The point: the electron is not "cancelling its radiation geometrically while orbiting". **In a stationary state it is not orbiting at all** — the classical picture of an orbit does not survive in quantum mechanics. **Bohr's circular orbit was a provisional hypothesis of 1913, superseded by the Schrödinger equation.**

Consistency with the mode criterion of Paper 14 (computed here): the field at the Bohr radius is

$E = e/(4\pi\epsilon_0 a_0^2) = 5.142 \times 10^{11}$ V/m, the Schwinger limit is $E_S = m_e^2 c^3 / (e\hbar) = 1.323 \times 10^{18}$ V/m, and the ratio is 3.886×10^{-7} (agreeing with the 3.9×10^{-7} in the second edition's table). The atomic scale therefore lies in **the ordinary regime**, not one where extreme-field arguments are needed.

7. Generalising it into a check

Corollary 7.1 (The series' second check)

The 3rd edition of Paper 11 set out the check that **"a quantity entering a normalised probability only as an overall factor cannot be observed, by definition"**. This paper places a second one beside it.

A convention must be a number. When one says that a quantity has been "fixed by convention", **check whether its value depends on the configuration (position, time, state)**. If it does, that is not a convention but **a change of the theory**.

Both checks were extracted from the author's own failures. In Paper 11 **a quantity that cancels was called a mechanism**. Here **a quantity that varies was called a constant**.

Observation 7.2 ("The backbone infrastructure of the universe's OS" is Layer 3)

Section 5 of the second edition stated that this paper's equation **"connects perfectly with Papers 14, 16 and 09, forming the backbone infrastructure network of the universe's OS"**. Paper 82 explicitly quarantined this kind of language (OS, rendering, infrastructure) in Layer 3, the non-scientific layer. This paper follows that discipline and removes the sentence from the body.

The phrase "connects perfectly" is withdrawn as well. That four papers contain the same 4π or $16\pi^2$ is not evidence of a common origin (Papers 48, 86, 100, 115).

Personal reflection of the author (Layer 3; labelled non-scientific / no claim of mechanism)

In the second edition I withdrew the "non-collapse theorem", because I had called $16\pi^2/16\pi^2 = 1$ a mechanism. I thought that settled it.

Going back over it for the 3rd edition I found that **I had not withdrawn enough**. The extension to n had never been derived, and "perfectly explains the Balmer series" had been left standing. **Withdrawing one thing leaves you feeling that the rest has been fixed too** — the same thing happened in Paper 06.

What told most was **putting $\varepsilon_0 = r$ literally into the force law**. It becomes an inverse cube. An inverse cube has no stable circular orbit. **The patch I applied in order to protect the atom will, read literally, drop it.**

I had grouped $m = 4\pi$ and $\varepsilon_0 = r$ under the one word "Yamagishi patch". One is a choice of units; the other is a function of the configuration. **Giving them the same name is what hid the distinction** — the same shape of failure as the one the series has kept warning about, "do not call the same number the same root". **A name can conceal a distinction.**

8. Scope / Conclusion

What is shown: that $m = 4\pi$ is a choice of units while $\varepsilon_0 = r$ is not, and that a convention must not be a function of the configuration (§2, the three roots of Paper 38) / that read as a force law, $\varepsilon_0 = r$ gives an inverse cube and destroys any stable circular orbit (§3, outside Bertrand's theorem) / that no n appears in the patched equation, so the extension to general quantum number is incompatible with quantisation (§4) / that it does hold as a restatement at one radius, and that in the implied system of units charge is an area per unit time (§5) / that atomic stability is already explained by quantum mechanics (§6) / the check "a convention must be a number" (§7).

What is not shown (restraint): no explanation of atomic stability, none of spectra, no replacement of quantum mechanics — all outside the scope of this paper / **Theorem 3.1 of the second edition and the claim to explain the Balmer series are withdrawn** / $(4\pi v)r = e$ is not extended beyond a single radius / "the backbone infrastructure of the universe's OS" is removed from the body (Layer 3; Paper 82) / no new physical law and no prediction (Layer 2).

What the 3rd edition adds is a way of judging the standing of a patch. A convention must be a number — the moment a configuration-dependent quantity is called a constant, it ceases to be a convention and becomes a change of the theory, and **the changed theory may explain less than the original.**

What survives is small but certain.

At one radius, the Bohr balance condition can be written $(4\pi v)r = e$.

This is an observation about notation concerning known physics, not a new explanation.

Revision Record

3rd edition (August 2026): removed the word "Withdrawal" from the title, which now names **what the paper shows**; the list of withdrawals is collected in this section.

Added in the 3rd edition: an account, against the three roots of Paper 38, of why $m = 4\pi$ (a choice of units) and $\varepsilon_0 = r$ (a function of the configuration) are not operations of the same kind (§2). A demonstration that read as a force law, $\varepsilon_0 = r$ gives an inverse cube whose effective potential has no minimum, so that no stable circular orbit exists, and that this lies outside Bertrand's theorem (§3). A dimensional analysis showing that in the system of units the equation implies, charge is an area per unit time (§5). The check "a convention must be a number", set beside the overall-factor check of Paper 11 (3rd ed.) (§7). The classical collapse time 1.556×10^{-11} s, the Bohr radius and velocity, the fine structure constant, and the ratio 3.886×10^{-7} of the atomic field to the Schwinger limit were **all computed here** (no external tables consulted).

Withdrawn in the 3rd edition:

1. **Theorem 3.1 of the second edition** (the extension $16\pi^2 v_n^2 r_n^2 = e^2/n^2$). No n appears in the patched balance, and compatibility with the quantisation condition $4\pi v_n r_n = n\hbar$ would require $e = n\hbar$ for every n , a contradiction (§4).
2. **"The discrete spectral structure of the Balmer series and the like is perfectly explained"**. The theorem it presupposed cannot be derived, and even if it held, the origin of the discreteness would remain Bohr's postulate — a change of notation, not an added explanation (§4).
3. **"Forming the backbone infrastructure network of the universe's OS" and "connects perfectly"** (§5 of the second edition). The former is Layer-3 vocabulary and has been removed from the body (Paper 82). The latter treats the same number as evidence of a common origin, contrary to the discipline of this series (Papers 48, 86, 100, 115).

Withdrawn in the 2nd edition (July 2026): the first edition presented, as a "microscopic non-collapse theorem", $\text{Ratio} = 16\pi^2/16\pi^2 = 1$, and stated that "since the cancellation ratio is exactly 1, radiative decay vanishes". That claim was withdrawn in full. Any non-zero number divided by itself gives 1; this is no property peculiar to $16\pi^2$. The same error appears in the first editions of Papers 09 ($16\pi^2/(4\pi)^2 = 1$) and 13, both withdrawn. The 3rd edition of Paper 11 recast this type as a general proposition: the error of calling a quantity that cancels a mechanism.

First edition: Zenodo 10.5281/zenodo.21600352.

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