

A Two-Set Cover Returns the Wrong Answer for the Circle

— Descent, Good Covers, and Why the Two-Patch Construction Really Worked — [Paper 151]

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— *From thinking to syncing.* —

ABSTRACT

This paper applies a brake to Paper 146. No conclusion is withdrawn. Paper 146 wrote the monopole on two patches and derived $2eg/\hbar = n$ as the condition that the gluing close up. **That integer is right. What was not right is the account of why two patches sufficed. NO NEW MATHEMATICAL THEOREM IS CLAIMED.** First, if the cover is not good, gluing returns the wrong answer. Covering the circle by two arcs gives a nerve that is a segment, returning $H^1 = 0$ — where the true answer is $\mathbb{R}$. Three arcs give the boundary of a triangle and return $H^1 = \mathbb{R}$ (computed here from boundary matrices) (Section 3). Second, three sets can never see the sphere. Enumerating every simplicial complex on three vertices (there are 18) shows that all of them have $H^2 = 0$ (computed here). Four sets, the boundary of a tetrahedron, first give $H^2 = \mathbb{R}$, and S^n needs $n + 2$ (checked for $n = 1, \dots, 4$) (Section 4). Third, and this is the core. Paper 146's two patches sat on a cover for which Čech cohomology returns the wrong answer. The overlap of two enlarged hemispheres is an annulus and not contractible, so the cover is not good. What was working is a different machine, clutching: a bundle built from two contractible patches is classified by the homotopy class of the transition function on the overlap, and that is $\pi_1(U(1)) = \mathbb{Z}$. Recounting the winding number reproduces $2eg/\hbar$ (checked here from -2 to 3) (Section 5). Fourth, the cocycle condition bites only from three sets onward. With two patches there are no triple overlaps and the condition is automatic — so no violation can be exhibited either. With three, $(1, 1, 1)$ fails to glue while $(1, -1, 0)$ and $(2, 3, -5)$ glue (Section 6).

Scope of this paper: NO NEW MATHEMATICAL THEOREM IS CLAIMED. Leray's theorem (the nerve of a good cover), the clutching construction and the cocycle condition of descent are all standard. What this paper adds is only exhibiting in numbers, in the smallest example, that a bad cover returns a wrong answer, settling “three sets cannot see the sphere” by a finite enumeration, and identifying which machine made Paper 146's two patches work. Paper 146's conclusion ($2eg/\hbar = n$) is not withdrawn. It is not claimed that magnetic monopoles exist.

1. Notation and position

The **nerve** of an open cover has one vertex per set and a simplex for each finite subfamily with non-empty intersection. A **good cover** is one in which every finite intersection is empty or contractible.

Position: Paper 146 gave an obstruction an address; Paper 147's separator was the coefficients, Paper 148's the test object, Paper 149's orientation and Paper 150's weight. **The separator here is the goodness of the cover — and it lands on the corpus itself.**

2. The machinery, checked

Verification 2.1: The procedure computing Betti numbers from the ranks of boundary matrices was checked on complexes whose answers are known.

complex	computed here	known
a point	(1)	(1)
a segment (two vertices, one edge)	(1, 0)	(1, 0)
the boundary of a triangle	(1, 1)	(1, 1)
a filled triangle	(1, 0, 0)	(1, 0, 0)
the boundary of a tetrahedron	(1, 0, 1)	(1, 0, 1)

(QED) Every computation below uses this procedure.

3. A two-set cover returns the wrong answer for the circle

Verification 3.1

cover	nerve	H^1 of the nerve	true H^1 of the circle
two arcs	a segment	0	$\mathbb{R}$
three arcs	the boundary of a triangle	$\mathbb{R}$	$\mathbb{R}$

(QED)

Observation 3.2

Each of the two arcs is contractible, but their intersection falls into two components. That is why the cover is not good. And a cover that is not good returns 0 where the answer is $\mathbb{R}$.

Observation 3.3

“Glue the local data and you recover the global” is not unconditional. What Leray's theorem demands is that the intersections be contractible.

4. Three sets can never see the sphere

Verification 4.1: Enumerating the simplicial complexes on three vertices (downward-closed families of faces) gives **18 of them**, and every one has $H^2 = 0$. (QED)

Observation 4.2

So no cover by three sets has a nerve that can see the sphere. This is not “no clever triple has been found” but a claim settled by a finite check.

Verification 4.3

sphere	sets	Betti numbers of the nerve
S^1	3	(1, 1)
S^2	4	(1, 0, 1)
S^3	5	(1, 0, 0, 1)
S^4	6	(1, 0, 0, 0, 1)

(QED) A good cover of S^n needs $n + 2$ sets, the nerve being the boundary of a simplex.

5. Why Paper 146's two patches really worked

Observation 5.1 (first, the cover is not good)

Paper 146 covered the sphere with a **northern and a southern patch**. The nerve of two sets is a segment and returns $H^2 = 0$ — the wrong answer for the sphere, for the reason of Verifications 3.1 and 4.1. **Indeed the overlap of two enlarged hemispheres is an annulus and is not contractible.**

Observation 5.2 (so what was working)

Clutching. A bundle built from two contractible patches is classified by **the homotopy class of the transition function on the overlap**. The overlap is an annulus, homotopy equivalent to a circle, so the set of classes is $[S^1 \rightarrow U(1)] = \pi_1(U(1)) = \mathbb{Z}$.

Verification 5.3: Counting numerically the winding number of Paper 146's transition function $\exp(2ieg\varphi/\hbar)$,

$2eg/\hbar$	winding number (computed here)
−2	−2.000000
−1	−1.000000
0	0.000000
1	1.000000
2	2.000000
3	3.000000

(QED) The invariant is the winding number, an element of π_1 of the overlap's homotopy type.

Observation 5.4 (what survives and what is corrected)

Paper 146's conclusion $2eg/\hbar = n$ is right and is not withdrawn. What is corrected is the single line saying that two patches suffice because the problem is that simple. Two patches suffice because the clutching machine needs only the overlap — and that same cover is precisely one on which Čech would lie.

Observation 5.5 (a familiar shape)

The reason that produced a right number belonged to a different machine — the same shape as Paper 149’s “it was not |det| but the largest invariant factor”. A misattributed reason cannot be caught while the answer comes out right.

6. The cocycle condition bites from three sets onward

Observation 6.1

With two patches there are no triple overlaps, so the condition is only $g_{12}g_{21} = 1$, which holds by definition. Nothing can break, and nothing can be tested.

Verification 6.2: Covering the circle by three arcs with integer (additive) transition data,

(g_{12}, g_{23}, g_{31})	sum	verdict
$(1, -1, 0)$	0	glues
$(1, 1, 1)$	3	does not glue
$(2, 3, -5)$	0	glues

(QED)

Observation 6.3

The obstruction to descent appears only from the third set onward. With two, it can neither be exhibited nor ruled out.

7. The census

Observation 7.1

machine	what it demands	where the answer lives	correct?
Cech with a good cover	finite intersections contractible	Leray	yes
Cech with any cover	nothing	—	no (Section 3)
clutching with two patches	both patches contractible	π_1 of the overlap	yes
descent with three or more	the cocycle condition on triples	Section 6	if it holds

Observation 7.2

“From local to global” is not one operation. Which machine is in use decides the demand, the home of the answer, and the reason it is right (Paper 100).

8. Conclusion

1. A cover that is not good returns a wrong answer (Ver. 3.1): two arcs return 0 for the circle where the answer is $\mathbb{R}$.
2. Three sets cannot see the sphere (Ver. 4.1): all 18 complexes on three vertices have $H^2 = 0$; S^n needs $n + 2$ (Ver. 4.3).
3. Paper 146’s two patches sat on a cover for which Cech returns the wrong answer (Obs. 5.1). What was working is clutching, and the invariant is the winding number (Ver. 5.3).
4. Paper 146’s conclusion is not withdrawn (Obs. 5.4); what is corrected is the line about why two patches suffice.
5. The cocycle condition bites from three onward (Ver. 6.2); with two, a violation cannot even be constructed.
6. “From local to global” is not one operation (Obs. 7.1).

Closing: Two patches sufficed not because the problem was simple. They sufficed because the machine in use needed only the overlap. And that same cover, handed to a different machine, lies. Change the tool and the answer changes on the same picture — which is why one must write down what was used.

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