

A Census of the Twos

— What Must Be Moved for Each 2 to Move — [Paper 157]

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— *From thinking to syncing.* —

ABSTRACT

This corpus has met the exponent 2 in many places. Each has been written about separately, but no paper has yet called them all by name at once. This one does, using a single test: what must be moved for that 2 to move? **NO NEW MATHEMATICAL THEOREM AND NO NEW PHYSICAL LAW IS CLAIMED.** First, the 2 of the inverse square is $d - 1$. Solving for spherically symmetric harmonic functions in d dimensions, the exponent of the force is 1, 2, 3, 4, 5 at $d = 2, \dots, 6$. Move the dimension and it moves (Section 2). Second, Born's 2 does not move with the dimension, and what fixes it is unitarity. The quantity $\sum_i |\psi_i|^p$ is preserved by a unitary only at $p = 2$: at $p = 1, 1.5, 2.5, 3, 4$ the change runs from 0.18 to 0.67, while at $p = 2$ it is 8.882×10^{-16} (computed here). Changing the dimension of the Hilbert space to 2, 3, 5, 8, 13 leaves it at order 10^{-15} (Section 3). Third, Itô's 2 moves with the roughness of the path. The quadratic variation should scale as n^{1-2H} , and the measured slopes at $H = 0.2, 0.35, 0.5, 0.65, 0.8$ were 0.5965, 0.3028, 0.0046, -0.3081 , -0.5917 — agreeing with $1 - 2H$ to within 0.01 (computed here) (Section 4). Fourth, and this is what is new here. The 2 of least squares is the assumed error law itself. Minimising $\sum |r_i|^p$ is maximum likelihood for a density proportional to $e^{-|r|^p}$, and the exponent that wins matches the exponent of the law that generated the data: $p = 2$ under Gaussian errors (0.1565 against 0.1983), $p = 1$ under Laplace errors (0.1782 against 0.2199), and under Cauchy errors $p = 1$ gives 0.2632 while $p = 2$ gives 128.3741 — a factor of about 500 (Section 5). Fifth, Clifford's 2 cannot be moved at all. The relation $v^2 = Q(v)$ is the definition of the algebra, and there is no parameter to turn (Section 6). One question, three answers, and three cases of “nothing at all” (Section 7).

Scope of this paper: NO NEW MATHEMATICAL THEOREM AND NO NEW PHYSICAL LAW IS CLAIMED. The inverse-square law, the Born rule, Itô's lemma, maximum likelihood and Clifford algebras are **all standard**. **Nor does this paper retract anything the corpus has said about the individual twos** — Paper 53 wrote that Born's exponent comes from two distinct roots, and Paper 132 that Itô's **2** is not a dimension. **What is added is (i) calling five twos by name under one test, (ii) showing in numbers that unitarity selects $p = 2$ uniquely, (iii) measuring the slope of the quadratic variation against $1 - 2H$, and (iv) giving the 2 of least squares its address as an assumed error law, exhibited as a factor of 500 under Cauchy noise.**

1. Notation and position

One test runs through the paper: **what has to be moved for this 2 to move?** If something is found, that is where the **2** lives. If nothing is, then either a definition or a uniqueness theorem is holding it.

Position: Paper 86 called the four roots of necessity, and Paper 100 the crossing of royal roads. Paper 117 showed **one phenomenon (scale invariance) fixing an exponent in four different ways**. This paper runs that in reverse: **one number (2), and how many different places it comes from.**

2. The 2 of the inverse square is $d - 1$

Verification 2.1: A spherically symmetric harmonic function in d dimensions satisfies $f'' + \frac{d-1}{r} f' = 0$. Solving and reading off the force,

d	potential	force	exponent
2	$C_1 + C_2 \log r$	r^{-1}	1
3	$C_1 + C_2/r$	r^{-2}	2
4	$C_1 + C_2/r^2$	r^{-3}	3
5	$C_1 + C_2/r^3$	r^{-4}	4
6	$C_1 + C_2/r^4$	r^{-5}	5

(QED)

Observation 2.2

The 2 of the inverse square is $d - 1$ read at $d = 3$. What must be moved is the spatial dimension (Papers 01 and 156).

3. Born's 2: unitarity selects it uniquely

Verification 3.1: Draw a random unitary U of size n and a random unit vector ψ , and see whether $\sum_i |\psi_i|^p$ changes under U (worst case over 400 draws at $n = 6$).

p	largest change in $\sum \psi_i ^p$
1	6.698×10^{-1}
1.5	2.279×10^{-1}
2	8.882×10^{-16}
2.5	1.798×10^{-1}
3	4.248×10^{-1}
4	4.364×10^{-1}

(QED)

Verification 3.2: Holding $p = 2$ and changing the dimension,

n	largest change
2	9.992×10^{-16}
3	7.772×10^{-16}
5	8.882×10^{-16}
8	8.882×10^{-16}
13	7.772×10^{-16}

(QED)

Observation 3.3

Only $p = 2$ is preserved, and that fact does not depend on the dimension. What must be moved is not the dimension but the demand that time evolution be unitary.

Observation 3.4

This paper does not call that a derivation of the Born rule. The derivation is Gleason's theorem (Paper 71), whose assumptions are far weaker than a random trial. What is shown here is only the step before: with any other exponent, probability is not conserved at all.

4. Itô's 2: what must be moved is the roughness

Verification 4.1: Generate fractional Brownian motion of Hurst exponent H exactly and measure the quadratic variation on $[0, 1]$ as the mesh is refined. Theory gives a scaling of n^{1-2H} .

H	$\sum(\Delta X)^2$ at $n = 2^{10}, 2^{12}, 2^{14}, 2^{16}$	slope	$1 - 2H$	verdict
0.2	64.700, 145.476, 334.388, 771.699	0.5965	0.60	diverges
0.35	7.866, 12.208, 18.386, 27.804	0.3028	0.30	diverges
0.5	0.979, 0.998, 0.988, 1.004	0.0046	0.00	finite, non-zero
0.65	0.129, 0.083, 0.055, 0.036	-0.3081	-0.30	vanishes
0.8	0.015, 0.007, 0.003, 0.001	-0.5917	-0.60	vanishes

(QED)

Observation 4.2

The measured slopes track $1 - 2H$ to within 0.01. The quadratic variation is finite and non-zero only at $H = 1/2$. What must be moved is the roughness of the path, not the spatial dimension (Paper 132).

5. The core: the 2 of least squares is an assumed error law

Fact 5.1: Minimising $\sum_i |r_i|^p$ is maximum likelihood for errors with density proportional to $e^{-|r|^p}$. $p = 2$ corresponds to the Gaussian and $p = 1$ to the Laplace law. So p is not a property of the data but the name of an assumption.

Verification 5.2: With a true location of 3.0, draw 40 points 2000 times and measure the root-mean-square error of the $p = 1$ and $p = 2$ estimators.

errors actually drawn from	error at $p = 1$	error at $p = 2$	which wins
Gaussian e^{-r^2}	0.1983	0.1565	$p = 2$
Laplace $e^{- r }$	0.1782	0.2199	$p = 1$
Cauchy (heavy tailed)	0.2632	128.3741	$p = 1$

(QED)

Observation 5.3

The exponent that wins matches the exponent of the law that generated the data. The 2 of least squares is not a property of “error” in general but precisely the fact that a Gaussian was assumed.

Observation 5.4

Under Cauchy noise the gap is about a factor of 500. That is not a loss of accuracy but a breakdown of the estimator — the Cauchy law has no variance, so minimising a sum of squares has lost its meaning. It is the same shape of failure that Paper 129 described when it wrote that the variance form of the uncertainty relation says nothing once the variance diverges.

Observation 5.5

So the 2 of least squares has a distinct root from the inverse square, from Born and from Itô. What must be moved is neither the dimension nor the roughness but the probability law one assumed.

6. Clifford's 2 cannot be moved

Verification 6.1: The defining relation of a Clifford algebra is $v^2 = Q(v)$. In the Pauli algebra,

$\sigma_x^2 - I$	$[0\ 0; 0\ 0]$
$\sigma_y^2 - I$	$[0\ 0; 0\ 0]$
$\sigma_z^2 - I$	$[0\ 0; 0\ 0]$

(QED)

Observation 6.2

There is no parameter here to turn. Replacing the 2 does not give a different Clifford algebra; it gives no Clifford algebra (the same shape as “ $\epsilon^2 = 0$ is a definition of a ring” in Papers 104 and 148).

7. The census

Observation 7.1

the 2	what moves it	general form	address
the inverse square	the spatial dimension d	$d - 1$	harmonicity (01, 156)
Born's	nothing (unitarity fixes it)	2	53, 68, 71
Itô's	the roughness H	finite only at $H = 1/2$	132, 148
least squares	the assumed error law	the p of $e^{- r ^p}$	this paper
Clifford's	nothing (a definition)	2	104, 148
Fourier's $1/2$	nothing (an extremal value)	$1/2$	129

Observation 7.2

One question, three answers, and three cases of “nothing at all”. What must be moved is the dimension, the roughness or the assumed law — or else it does not exist.

Observation 7.3

The three “nothings” are not the same either. Born's 2 is held by a uniqueness theorem, Clifford's by a definition, and Fourier's $1/2$ arrives as the value of an extremum. The verdict is the same and the reason for it is different in all three.

8. Conclusion

1. The inverse-square 2 is $d - 1$ (Ver. 2.1): it moves with the dimension.
2. Only $p = 2$ conserves $\sum |\psi|^p$ (Ver. 3.1): 8.882×10^{-16} against order 10^{-1} .
3. That $p = 2$ is dimension-independent (Ver. 3.2): order 10^{-15} from $n = 2$ to 13.
4. The slope of the quadratic variation tracks $1 - 2H$ (Ver. 4.1): finite only at $H = 1/2$.
5. The 2 of least squares is an assumed error law (Ver. 5.2): 0.2632 against 128.3741 under Cauchy.
6. Clifford's 2 cannot be moved (Ver. 6.1).

Closing: One shape of exponent, arriving from this many different places. On meeting a 2, the question to ask is not “why two?” but what would have to move for it to stop being two. Only when that question has no answer may the 2 be written down as something that does not move.

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