

The Equals Sign Has Distinct Roots Too

— 3rd Edition: What the Yamagishi Full-Synchronisation Equation Was Chaining — [Paper 16]

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August 2026 (3rd Edition)

— *From thinking to syncing.* —

ABSTRACT

The first edition of this paper presented the one-line chain $p = 4\pi v = e/r = c\sqrt{n_{\text{opt}}}$ and claimed that four domains of physics were thereby "fully synchronised". The second edition found that **the three equals signs have different logical standings** and withdrew the "grand unification" claim. **The 3rd edition turns that finding into a census the series can use. First, the equals sign has distinct roots too.** "=" is not one kind of assertion. This paper takes a census of **six roots**: **(a) definition** (true by stipulation, cannot be false), **(b) consequence of substitution** (follows automatically once a convention is adopted), **(c) identity** (true for all values), **(d) conditional** (holds only on solutions), **(e) empirical law** (could have been false, and has been tested), **(f) dimensionally void** (not even a candidate for truth). **Writing them all with the same "=" made the four quantities look as though they had the same standing. Second, this paper's chain was joined by three different roots.** $p = 4\pi v$ is (b); $4\pi v = e/r$ is (d) (centripetal balance on a circular orbit, not valid for arbitrary v, r); $e/r = c\sqrt{n_{\text{opt}}}$ is (f). What looked like one chain was **three different kinds of relation set side by side. Third, the dimensional mismatch the second edition recorded as "unresolved" does resolve — but the way it resolves is the problem.** In the system of units identified in the 3rd edition of Paper 15 ($\varepsilon_0 = r$ so $[\varepsilon_0] = L$; $m = 4\pi$ so $M = 1$; hence $C = L^2T^{-1}$), all four quantities have dimension L/T . But the price of that system is $\varepsilon_0 = r$, which Paper 15 showed is not a convention at all. The agreement of dimensions is not evidence for the chain; it is a **consequence of the same move. Fourth, the third equals sign is not "future work".** The standard optical relation in which a square root of the refractive index arises is **Maxwell's relation** $n = \sqrt{\varepsilon_r \mu_r}$. Hence $\sqrt{n_{\text{opt}}} = (\varepsilon_r \mu_r)^{1/4}$ — a quantity that does not exist in optics. Since the square root has no source, this is not an unfinished equation but a **rootless one. Fifth, what remains is that one substitution appears in several places.** $E_k = 2\pi v^2$ (Paper 18), $P = \frac{4}{3}\pi n v^2$ (Paper 19) and $kT = \frac{4}{3}\pi v^2$ (Paper 20) are all just $m = 4\pi$ put into a standard formula. And that 4π is the "choice of units" 4π of Paper 35 (3rd ed.), not the "forced" 4π of Gauss's flux. A unification built on a choice of units unifies the notation. Scope: a structural account within established physics (Layer 2). **This paper claims no unification of physical domains.**

Positioning of this paper (discipline): This paper offers a **structural interpretation** of established physical laws. It claims neither the discovery of new physical laws nor the proof of open problems. The relations shown are transformations of known expressions; the aim is to propose a structural reading of them. No claim contradicting established theory is made. **The appearance of the same number in several contexts is not treated as evidence of a common origin** (Papers 48, 86, 100, 115). **$m = 4\pi$ and the like are conventional choices within a system of units, not new facts about nature** (Papers 35, 38, 95).

1. Introduction: the four quantities this paper chained

The first edition presented the following line and claimed that mechanics, electromagnetism, quantum mechanics and wave optics were thereby "fully synchronised":

$$p = 4\pi v = \frac{e}{r} = c\sqrt{n_{\text{opt}}} \quad (\text{1st ed.})$$

The second edition withdrew the "grand unification" claim and recorded that **the three equals signs have different logical standings. That observation was right, but it stopped there.**

The work of the 3rd edition is to turn that observation into a census the series can use.
The 3rd editions of Papers 11 and 15 each extracted a check from a failure. This paper extracts a third.

2. The equals sign has distinct roots too — a census of six

Observation 2.1 ("=" is not one kind of assertion)

Paper 112 took a census of **six distinct roots** beneath the single intuition "**a region is allotted, and within it one is free**". This paper does the same beneath the single symbol "**=**".

root	kind of equality	example and character
(a)	definition	$p \equiv mv$. True by stipulation; cannot be false .
(b)	consequence of substitution	With $m = 4\pi$, $p = 4\pi v$. Automatic once adopted.
(c)	identity	$\sin^2 \theta + \cos^2 \theta = 1$. True for all values.
(d)	conditional	$mv^2/r = e^2/4\pi\epsilon_0 r^2$. Holds only on solutions.
(e)	empirical law	$v_{\text{phase}} = c/n_{\text{opt}}$. Could have been false; tested.
(f)	dimensionally void	The sides differ in dimension. Not a candidate at all .

Corollary 2.2 (There is an ordering of content among the six)

(a) and (b) say nothing about the world — the first is a naming, the second a consequence of a naming. (c) is said by mathematics; (d) and (e) are said by the world. And (f) says nothing at all.

Chaining quantities with "=" therefore makes them look as though they had the same standing. The symbol hides the difference in standing — the same shape as the 3rd edition of Paper 15, which wrote that "**a name can conceal a distinction**". There it was a name; here it is a symbol.

3. Taking the census of this paper's own chain

Observation 3.1 (What looked like one chain was three different kinds of relation)

equality	root	content
$p = 4\pi v$	(b)	Follows once $m = 4\pi$ is adopted. Not a discovery.
$4\pi v = e/r$	(d)	Centripetal balance (Paper 15). Not valid for arbitrary v, r .
$e/r = c\sqrt{n_{\text{opt}}}$	(f)	Dimensions disagree; $\sqrt{n_{\text{opt}}}$ has no source (§5).

The first edition's line therefore did not join four quantities in one chain. It set three different kinds of relation side by side under the same symbol.

Note especially the character of (d). As the second edition correctly recorded, $4\pi v r = e$ holds only when the electron is in a particular bound orbit. The 3rd edition of Paper 15 narrowed the range further — it holds at a single radius, and can neither be differentiated with respect to r nor extended to other radii. The middle link of the chain is joined only at a point.

4. The dimensional mismatch resolves — but the way it resolves is the problem

The second edition recorded as unresolved that "the four quantities do not agree in dimension in SI".

quantity	SI dimension
p	kg·m/s
$4\pi v$	m/s
e/r	C/m
$c\sqrt{n_{\text{opt}}}$	m/s

Established Fact 4.1 (In the system of units of Paper 15, all four are L/T)

The 3rd edition of Paper 15 showed that $\epsilon_0 = r$ (hence $[\epsilon_0] = L$) together with $m = 4\pi$ (hence $M = 1$) forces $C = L^2T^{-1}$ (charge as area per unit time). In that system:

quantity	breakdown	dimension
$p = mv$	$M = 1$, so $(1)(L/T)$	L/T
$4\pi v$	4π dimensionless	L/T
e/r	$C/L = (L^2T^{-1})/L$	L/T
$c\sqrt{n_{\text{opt}}}$	n_{opt} dimensionless	L/T

All four agree at L/T . The problem the second edition recorded as unresolved does resolve.

Corollary 4.2 (But resolving it is not evidence)

The condition that buys this agreement is $\epsilon_0 = r$. And the 3rd edition of Paper 15 showed that $\epsilon_0 = r$ is not a convention — a convention must be a number, and something that assigns a different value at every radius is not a choice of units.

So "the dimensions agree" is not evidence for the chain; it is a consequence of the same single illegitimate move. Two results of one operation must not be used as evidence for each other.

An aside: writing "in natural units" does not by itself fix the equation either. In Heaviside–Lorentz units $\alpha = e^2/4\pi$, so $e = \sqrt{4\pi\alpha} = 0.3028$; in Gaussian units $\alpha = e^2$, so $e = \sqrt{\alpha} = 0.0854$ (computed here). The number changes with the choice.

5. The third equals sign is not "future work"

The second edition recorded $e/r = c\sqrt{n_{\text{opt}}}$ as "no derivation given; withdrawn and left as future work". The 3rd edition removes it from the list of tasks.

Established Fact 5.1 ($\sqrt{n_{\text{opt}}}$ is not a quantity in optics)

The standard relation in which a square root of the refractive index arises is **Maxwell's relation**

$$n_{\text{opt}} = \sqrt{\epsilon_r \mu_r},$$

so that

$$\sqrt{n_{\text{opt}}} = (\epsilon_r \mu_r)^{1/4}$$

— **the fourth root of $\epsilon_r \mu_r$** . Computed here: **1.2247** for $\epsilon_r = 2.25$ ($n = 1.5$) and **1.4142** for $\epsilon_r = 4$ ($n = 2$). **This quantity occurs in no standard relation of optics.**

The phase velocity in a medium, meanwhile, is $v_{\text{phase}} = c/n_{\text{opt}}$: **no square root, and n_{opt} in the denominator.**

Corollary 5.2 (Not unfinished — rootless)

Since no source for the square root can be identified, this is not "an equation whose derivation is incomplete" but "an equation with no root". Leaving it as future work implies that the gap will one day be filled. The 3rd edition removes that implication.

A caution about distinct roots for the exponent 1/4: a fourth root also appears in the 3rd edition of Paper 03 (the Kolmogorov length $\eta = (\nu^3/\epsilon)^{1/4}$), but **it is a distinct root**. There it is **forced by the determinant $|\det| = 4$ of the dimension matrix**; here it is **an ungrounded square root of a square root**. The two 1/4s must not be **joined** (Papers 48, 86, 100, 115).

6. What remains — one substitution appearing in several places

Observation 6.1 (Papers 18, 19 and 20 are three appearances of one substitution)

Verified here (simply substituting $m = 4\pi$ into a standard formula):

standard formula	with $m = 4\pi$	paper
$E_k = \frac{1}{2}mv^2$	$E_k = 2\pi v^2$	Paper 18
$P = \frac{1}{3}nm\langle v^2 \rangle$	$P = \frac{4}{3}\pi n v^2$	Paper 19
$\frac{3}{2}kT = \frac{1}{2}m\langle v^2 \rangle$	$kT = \frac{4}{3}\pi v^2$	Paper 20

All three are one substitution into a standard formula; no new relation appears. The second edition already withdrew the phrases "a master hub structure binding all 24 papers" and "governs them perfectly". What the 3rd edition adds is the explicit statement that this is not a "transfer of machinery" in the sense of Paper 100, but one substitution appearing in several places.

7. A unification built on a choice of units unifies the notation

Corollary 7.1 (Which root is this paper's 4π ?)

The 3rd edition of Paper 35 sorted the two 4π 's inside $4\pi G$: the 4π of Poisson's equation is forced by Gauss's flux and cannot be chosen, whereas the 4π of $G = 4\pi$ is a choice of units and can be.

The 4π of $p = 4\pi v$ is the latter. Being a chosen number, its appearance in several domains is no cause for surprise — the same number was put in four places, so it appears in four places.

A unification built on a choice of units unifies the notation.
It does not unify the physics.

Observation 7.2 (The series' third check)

The 3rd edition of Paper 11 set out "**a quantity entering a normalised probability only as an overall factor cannot be observed, by definition**", and the 3rd edition of Paper 15 set out "**a convention must be a number**". This paper places a third beside them.

Write out the root of each equals sign. When several quantities are chained with "=", **state for each equality whether it is (a) a definition, (b) a consequence of substitution, (c) an identity, (d) a conditional, (e) an empirical law, or (f) dimensionally void.** Without that, the symbol hides the difference in standing.

All three checks were extracted from the author's own failures. In Paper 11 **a quantity that cancels was called a mechanism**; in Paper 15 **a quantity that varies was called a constant**; here **different kinds of relation were set side by side under one symbol.**

Personal reflection of the author (Layer 3; labelled non-scientific / no claim of mechanism)

In the first edition I put four quantities on one line and wrote "grand unification". When the second edition withdrew it, I wrote that "**the three equals signs have different standings**" and was satisfied.

What I saw writing the 3rd edition is that **that one sentence could be the substance of the paper. The equals sign has distinct roots too.** The series has kept saying "do not call the same number the same root", but **the same applies to symbols.** I had learned to be wary of 4π ; **I had not learned to be wary of "=".**

And what stung most was that **the dimensions agreed.** Rewritten in the system of units of Paper 15, all four come out cleanly as L/T . **For a moment I felt saved.** But the condition buying that system was the very thing I had myself judged, in Paper 15, not to be a convention. **I was about to use two results of one operation as evidence for each other.**

The moment of feeling saved is the most dangerous one. That, I think, is what Paper 101 was writing about when it wrote about Eddington.

8. Scope / Conclusion

What is shown: that "=" has six distinct roots (§2: definition / consequence of substitution / identity / conditional / empirical law / dimensionally void) / that **this paper's chain set three different kinds of relation under one symbol** (§3) / that **the dimensional mismatch resolves in the system of units of Paper 15, but that this is a consequence of the same**

move rather than evidence (§4) / that $\sqrt{n_{\text{opt}}} = (\epsilon_r \mu_r)^{1/4}$ does not exist in optics, so the third equality is rootless rather than a task (§5, a distinct root from the $1/4$ of Paper 03) / that Papers 18, 19 and 20 are three appearances of one substitution (§6) / that this paper's 4π is on the "choosable" side, so a unification built on it unifies the notation (§7). What is not shown (restraint): no unification of physical domains, no grand unification, no master API — all withdrawn and outside the scope of this paper / $e/r = c\sqrt{n_{\text{opt}}}$ is not left as future work / the agreement of dimensions is not used as evidence for the chain / this paper's $1/4$ is not joined to the $1/4$ of Paper 03 / no new physical law and no prediction (Layer 2).

What the 3rd edition adds is a way of judging the standing of a symbol. Being wary of numbers is not enough. "=" too makes different things look the same.

And what remains is small.

With $m = 4\pi$ one may write $p = 4\pi v$ — a consequence of substitution, not a discovery.
That it appears in several papers is because the same substitution was made several times.

Revision Record

3rd edition (August 2026): removed the word "Withdrawal" from the title, which now names **what the paper shows**; the list of withdrawals is collected in this section.

Added in the 3rd edition: a census of the six distinct roots of the equals sign (definition / consequence of substitution / identity / conditional / empirical law / dimensionally void), and a sorting of this paper's own chain into three of them (§§2–3). A demonstration that the dimensional mismatch the second edition recorded as unresolved does resolve in the system of units of the 3rd edition of Paper 15 ($C = L^2 T^{-1}$), all four quantities coming out as L/T , together with the statement that this is a consequence of the same move rather than evidence (§4). A demonstration that $\sqrt{n_{\text{opt}}}$ is the fourth root of $\epsilon_r \mu_r$ under Maxwell's relation and does not exist in optics, so that the third equality is removed from the list of future tasks (§5, with the explicit note that it is a distinct root from the $1/4$ of Paper 03). A table showing that Papers 18, 19 and 20 are three appearances of one substitution (§6). An explicit statement that this paper's 4π is on the "choosable" side in the sense of the 3rd edition of Paper 35 (§7). The check "write out the root of each equals sign", set beside the checks of the 3rd editions of Papers 11 and 15 (§7). The value of e from α , the coefficients of Papers 18/19/20, and the values of $(\epsilon_r \mu_r)^{1/4}$ were all computed here (no external tables consulted).

Changed in the 3rd edition: the second edition recorded $e/r = c\sqrt{n_{\text{opt}}}$ as "withdrawn and left as future work". The 3rd edition removes it from the list of tasks — since no source for the square root can be identified, it is not an unfinished equation but a rootless one (§5).

Withdrawn in the 2nd edition (July 2026): the first edition called this line "**the complete proof of a one-line grand-unified master API**". That claim was withdrawn in full.

1. $p = 4\pi v$ is a trivial consequence of substitution, not a discovery.
2. $4\pi v = e/r$ is a dynamical condition, the centripetal balance on a circular orbit, not an identity, and **does not hold for arbitrary v, r** .
3. $e/r = c\sqrt{n_{\text{opt}}}$ was never derived, and $c\sqrt{n_{\text{opt}}}$ is not a standard optical quantity.
4. The four quantities do not agree in dimension in SI.
5. The phrases "**a master hub structure binding all 24 papers**" and "**governs them perfectly**" in Section 4 were withdrawn as well; what actually holds is **the consistency that the same substitution produces the same form**. The temperature of Paper 20 was corrected to $kT = \frac{4}{3}\pi v^2$ (the first edition wrote v^3), and two discarded drafts were removed from the references.

First edition: Zenodo 10.5281/zenodo.21600471.

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