

Three Ceilings over One Inequality

— 2 Is Combinatorics, $2\sqrt{2}$ Is Geometry, 4 Is Arithmetic — [Paper 160]

Yuuki YAMAGISHI (山岸 勇輝)

Yamagishi Complexity Torus Lab, Japan

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— *From thinking to syncing.* —

ABSTRACT

Three upper bounds are spoken of for the CHSH quantity S — the classical **2**, the quantum $2\sqrt{2}$, and the algebraic **4**. **This paper calls all three by name and shows that no two of them are held down by the same thing. NO NEW PHYSICAL LAW IS CLAIMED.** First, the classical ceiling is combinatorics. There are only **16** local deterministic strategies, and exhausting them puts S inside $\{-2, \dots, 2\}$ (computed here). Mixing cannot raise it, so $|S| \leq 2$ comes from counting sign patterns alone (Section 2). Second, the quantum ceiling comes out of density matrices. On a singlet with $a = 0^\circ$, $a' = 90^\circ$, $b = 45^\circ$, $b' = -45^\circ$ the four correlations are all ∓ 0.7071067811865475 and $S = 2.8284271247$ — differing from $2\sqrt{2}$ by 4.44×10^{-16} . Over 40000 random states and directions it was never once exceeded (largest found **2.7291936631**) (Section 3). Third, and this is the core. What stops quantum mechanics there is geometry. For the singlet $E(a, b) = -a \cdot b$, so $S \leq |b + b'| + |b - b'|$, and the maximum of that right side over two unit vectors is **2.8284271247** (checked over 200000 pairs, attained when they are orthogonal). From $|x|^2 + |y|^2 = 2$ it follows that $|x| + |y| \leq 2\sqrt{2}$ — the geometry of an inner product, not combinatorics (Section 4). Fourth, the brake. What forbids **4** is not relativity. The PR box has $E = (1, 1, 1, -1)$ and reaches $S = 4$ exactly. Its marginals stay at 0.500 whatever the other side's setting — the PR box does not signal (verified here). Relativity permits $S = 4$ (Section 5).

Scope of this paper: NO NEW PHYSICAL LAW IS CLAIMED. Bell's inequality, CHSH, the Tsirelson bound, the PR box and the no-signalling condition are **all standard**. Nor does this paper answer why nature stops at $2\sqrt{2}$ — that is an open research question, and what is written here is **only the address of each of the three ceilings**. What is added is **obtaining the classical ceiling by exhaustion, obtaining the quantum one from density matrices and confirming non-exceedance by random search, exhibiting the source of $2\sqrt{2}$ numerically as the geometry of an inner product, and checking that the PR box reaches 4 while not signalling, which fences the explanation that relativity is what stops us.**

1. Notation and position

Two parties each have two settings and outcomes ± 1 . Writing the correlations as $E(x, y)$, the CHSH quantity is

$$S = E(a, b) + E(a, b') + E(a', b) - E(a', b').$$

Position: Paper 68 showed that a qubit is a Hopf fibration, and Paper 71 placed the Born rule beneath Gleason's theorem. Paper 112 split “a frame is issued and inside it one is free” into six distinct roots and wrote that Bell's line is not conceded. This paper stands on that line and counts how many lines there are.

2. The classical ceiling comes from counting signs

Verification 2.1: A local deterministic strategy fixes four signs $A_1, A_2, B_1, B_2 \in \{\pm 1\}$ in advance; there are only 16. Exhausting them,

values S can take	a set with endpoints $\{-2, 2\}$ (all 16 cases)
maximum of $ S $	2

(QED)

Observation 2.2

Mixing cannot raise it. A probabilistic mixture is a convex combination of these extreme points, so the maximum is attained at one of them. So $|S| \leq 2$ follows from the assumption of local determinism alone, by counting signs — its address is combinatorics.

3. The quantum ceiling, from density matrices

Verification 3.1: Measure a singlet along $a = 0^\circ, a' = 90^\circ, b = 45^\circ, b' = -45^\circ$.

$E(a, b)$	-0.7071067811865475
$E(a, b')$	-0.7071067811865475
$E(a', b)$	-0.7071067811865475
$E(a', b')$	$+0.7071067811865475$
S	2.8284271247
$2\sqrt{2}$	2.8284271247
difference	4.44×10^{-16}

(QED)

Verification 3.2: Drawing both the two-qubit state and the four measurement directions at random, **40000** times,

largest $ S $ found	2.7291936631
Tsirelson bound $2\sqrt{2}$	2.8284271247
exceeded	never

(QED)

Observation 3.3

Random directions rarely land near the optimal configuration, which is why the search stopped at 2.729. All this paper takes from the trial is that the bound was not exceeded; it is not a proof of the bound — the proof is the geometry of the next section.

4. The core: what stops quantum mechanics there is geometry

Fact 4.1: For the singlet $E(a, b) = -a \cdot b$, hence

$$|S| = |a \cdot (b + b') + a' \cdot (b - b')| \leq |b + b'| + |b - b'|.$$

Verification 4.2: Maximise that right side over two unit vectors.

maximum over 200000 random pairs (b, b')	2.8284271247
at right angles $(b \cdot b' = 0)$	2.8284271247
$2\sqrt{2}$	2.8284271247

(QED)

Observation 4.3

The reason fits on one line. Setting $x = b + b'$ and $y = b - b'$ gives $|x|^2 + |y|^2 = 2(|b|^2 + |b'|^2) = 4$, and the sum of two numbers with a fixed sum of squares is largest when they are equal. Hence $|x| + |y| \leq 2\sqrt{2}$.

Observation 4.4

So the address of $2\sqrt{2}$ is the inner product — that quantum correlations are inner products of unit vectors is the whole of what fixes the ceiling. The Bloch-sphere geometry of Paper 68 appears here unchanged. Where the classical 2 counted signs, this measures lengths — distinct roots.

5. The brake: what forbids 4 is not relativity

Verification 5.1: Define the PR box by $P(ab|xy) = \frac{1}{2}$ when $a \oplus b = xy$ and 0 otherwise.

$E(0,0)$	+1
$E(0,1)$	+1
$E(1,0)$	+1
$E(1,1)$	−1
S	4

(QED)

Verification 5.2: Ask whether the box signals, by writing one party's marginal for each of the other's settings.

own setting x	$P(a = 0)$ when $y = 0$	when $y = 1$	equal?
0	0.500	0.500	yes
1	0.500	0.500	yes

(QED)

Observation 5.3

The marginals do not depend on the other side's setting. The PR box does not signal. Relativity therefore permits $S = 4$ — and the explanation “it stops at $2\sqrt{2}$ because of relativity” does not explain that.

Observation 5.4

What stops it is the state space. Locality was already surrendered at $S > 2$, and no-signalling allows all the way to 4. What remains is the single fact that the correlations are inner products of unit vectors, and that gives $2\sqrt{2}$.

Observation 5.5

This paper does not answer why nature chose that state space. Work deriving the Tsirelson bound from principles such as information causality continues, and this paper does not enter it — what can be written is the address, and no further (the same stance as Paper 112).

6. The census: three ceilings

Observation 6.1

ceiling	what holds it down	to move it	address
2	local determinism	allow non-locality	combinatorics
$2\sqrt{2} = 2.8284271$	the quantum state space	change the inner-product geometry	geometry (Paper 68)
4	outcomes drawn from ± 1	change the range of outcomes	arithmetic

Observation 6.2

One inequality carries three ceilings, and no two are held down by the same thing. The sentence “Bell's inequality has been violated” is a statement about the first ceiling only.

Observation 6.3

And this 2 and this $\sqrt{2}$ are distinct from all five twos counted in Paper 157. Here the 2 comes from arrangements of signs and the $\sqrt{2}$ from the sum of two numbers with a fixed sum of squares — the census of that one word is not finished.

7. Conclusion

1. The classical ceiling is 2 by exhaustion (Ver. 2.1): all 16 cases.
2. The quantum ceiling is 2.8284271247 (Ver. 3.1): 4.44×10^{-16} from $2\sqrt{2}$.
3. It was never exceeded in 40000 random trials (Ver. 3.2): though that is not a proof.
4. The source of $2\sqrt{2}$ is the geometry of an inner product (Ver. 4.2, Obs. 4.3).
5. The PR box reaches $S = 4$ and does not signal (Ver. 5.1, 5.2).
6. All three ceilings are held down by different roots (Obs. 6.1).

Closing: Once one notices that there are three ceilings, the word “violated” turns imprecise. What was violated is the first. The second has never once been broken, and the third stands empty in a place relativity allows, with nobody living there.

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