

Uniformization Gives Not One Shape but One Model and One Parameter

— The Plane and the Disk Are Homeomorphic and Not Conformal: One Real Number Topology Cannot See —

[Paper 167]

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— *From thinking to syncing.* —

ABSTRACT

The uniformization theorem says that every simply connected Riemann surface is the sphere, the plane or the disk. A split into three sounds like a statement of topology — but the plane and the disk are homeomorphic. What separates them is not topology. **NO NEW MATHEMATICAL THEOREM IS CLAIMED** — all of this is standard since Riemann. **First, a trap.** The map $h(z) = z/(1 - |z|)$ is a homeomorphism from the disk onto the plane, and it keeps the right angle between the radial and tangential directions at exactly 90.0 degrees. Preserving one angle is not conformality — tilted by 45 degrees the same right angle opens to 157.38 degrees at $z = 0.8$, and the ratio of the largest stretch to the smallest is 1.25, 2.0, 5.0 against 1.0 for a holomorphic map (Section 2). **Second, and this is the core.** All eccentric annuli are homeomorphic to one another, and two are conformally the same exactly when one real number agrees. That number was computed along two independent routes, an explicit Möbius construction and the $\operatorname{arccosh}$ formula, and the two agreed to 20 digits in six cases (Section 3). **Third, the family with one number was solved explicitly.** With $R = 1$ and offsets $d = 0.1, 0.2, 0.3, 0.4$, inner radii 0.296707, 0.286870, 0.270614, 0.248139 give four regions that look nothing alike and carry the identical modulus 1.203973 (Section 4). **Fourth, on a torus the parameter becomes complex.** Building j from theta functions independently gives $j(i) = 1728$ and $j(2i) = 287496$, and $j(\tau) = j(\tau + 1) = j(-1/\tau)$ was verified to 10^{-27} . Uniformization does not end in one shape (Section 5). **Fifth, the sign of χ decides only for closed surfaces.** The plane and the disk both have $\chi = 1$ and are both contractible, yet fall to different models — the type of an open surface is settled by no topological invariant at all (Section 6).

Scope of this paper: NO NEW MATHEMATICAL THEOREM IS CLAIMED — the uniformization theorem, Liouville's theorem, the modulus of an annulus, the j -invariant with $SL(2, \mathbb{Z})$, and the type problem are all standard. Nor does this paper prove uniformization — what is offered is a measurement, not a proof. What is added is computing the modulus along two independent routes and confirming that they agree, solving explicitly for a family of eccentric annuli that share one modulus, exhibiting in numbers the trap that one right angle can be preserved by a map that is not conformal, and setting side by side the fact that χ decides for closed surfaces and does not for open ones.

1. Notation and position

A map is **conformal** when it preserves the angle between **every** pair of directions at each point. Here the stretch is measured along **720** directions at each point and the ratio of the largest to the smallest is called the **distortion**; distortion **1** and conformality are the same condition. The **modulus** of an annulus is $\log(R/r)$ for the concentric annulus $r < |z| < R$ it maps to.

Position: Paper 111 wrote that light determines geometry only as far as the conformal structure, and Paper 105 showed in dimension four that topology and differentiable structure have distinct roots. This paper treats the layer between them — what topology does not separate and conformal geometry does.

2. The trap: preserving one right angle is not conformality

Fact 2.1: $h(z) = z/(1 - |z|)$ is a bijection from the open disk onto the plane, with inverse $w \mapsto w/(1 + |w|)$. The worst round-trip error over **16** sample points was 1.2×10^{-32} .

Verification 2.2: The angle between the images of the radial and tangential directions, which cross at a right angle.

point	under h	under $z \mapsto z^2 + z$
$z = 0.2$	90.0	90.0
$z = 0.5$	90.0	90.0
$z = 0.8$	90.0	90.0

Both preserve it. (QED)

Observation 2.3

A radial map always keeps this particular right angle — the radial and tangential directions stay orthogonal in the image. So “preserves angles” must never be checked on one pair of directions.

Verification 2.4: The same right angle tilted by 45 degrees, and the ratio of stretches over all 720 directions.

point	h : tilted angle	$z^2 + z$: same	h : distortion	$z^2 + z$: distortion
$z = 0.2$	102.68038	90.0	1.25	1.0
$z = 0.5$	126.8699	90.0	2.0	1.0
$z = 0.8$	157.38014	90.0	5.0	1.0

(QED)

Observation 2.5

The holomorphic map has distortion exactly 1.0 at every point. That of the homeomorphism grows towards the edge — it is $1/(1 - r)$. Topology does not see this number.

Observation 2.6

And no conformal one exists at all. A conformal bijection from the plane onto the disk would be a bounded entire function, and Cauchy's estimate $|f'(0)| \leq M/R$ holds for every R : at $R = 10, 10^3, 10^6, 10^{12}$ the bound falls to $0.1, 10^{-3}, 10^{-6}, 10^{-12}$, forcing $f' = 0$ (Liouville). There is no non-constant bounded entire function.

3. The core: one real number topology cannot see

Notation 3.1: Consider the region between the circle $|z| = R$ and the circle $|z - d| = r$. Any two such regions are homeomorphic.

Verification 3.2: The modulus was computed along two independent routes. The first solves for the two points p, q inverse with respect to both circles ($pq = R^2$ and $(p - d)(q - d) = r^2$), builds the Möbius map $w = (z - q)/(z - p)$,

confirms at **400** points on each circle that the images **really are concentric circles**, and then reads the ratio of radii. The second is $\operatorname{arccosh}\left(\frac{R^2+r^2-d^2}{2Rr}\right)$.

R	r	d	via Möbius	via arccosh	spread of image radii
1	0.3	0	1.203972804	1.203972804	(already concentric)
1	0.3	0.2	1.15881036	1.15881036	9.9×10^{-32}
1	0.3	0.5	0.8670147265	0.8670147265	3.9×10^{-31}
1	0.5	0.4	0.4211448485	0.4211448485	5.4×10^{-31}
2	0.1	1.5	2.161580941	2.161580941	1.1×10^{-30}
1	0.45	0.5	0.3399260616	0.3399260616	9.9×10^{-31}

In all six the two routes agreed to 20 digits, and the image circles were concentric to within numerical dust. (QED)

Observation 3.3

The $\log(R/r)$ of the concentric case is replaced by an $\operatorname{arccosh}$ in the eccentric one — and since $\cosh(\log(R/r)) = (R^2 + r^2)/(2Rr)$ at $d = 0$, the two are one formula read twice.

4. Regions that look different and carry the same number

Verification 4.1: Fix $R = 1$ and solve for the inner radius that gives modulus **1.203972804**, the value of the concentric annulus with $r = 0.3$.

offset d	inner radius r	modulus, remeasured via Möbius
0	0.3	1.203972804
0.1	0.296706871877	1.203972804
0.2	0.286870020776	1.203972804
0.3	0.270614346149	1.203972804
0.4	0.248139402709	1.203972804

(QED)

Observation 4.2

Drawn on paper the five regions look nothing alike — one has its hole in the middle, another has it pushed towards the rim. They are conformally the same region. And the region with $R = 1$, $r = 0.45$, $d = 0.5$, of modulus 0.3399, is a different one.

Observation 4.3

One real number decides, and topology sees none of it. This runs opposite to Paper 152, where the same character table left two groups non-isomorphic — there, one invariant and different objects; here, different appearances and one object. In both, the answer changes with the invariant one asked by.

5. Tori: the parameter becomes complex

Fact 5.1: A torus is $\mathbb{C}$ modulo a lattice, and two are conformally equivalent exactly when their τ agree modulo $SL(2, \mathbb{Z})$.

Verification 5.2: Here j was built from theta functions as $\lambda = \theta_2^4/\theta_3^4$ and $j = 256(1 - \lambda + \lambda^2)^3/(\lambda^2(1 - \lambda)^2)$, so that no normalisation is taken on trust, and checked against a second route.

$j(i)$ from theta functions	1728.0
$j(i)$ by the second route	1728.0
$ j(e^{2\pi i/3}) $	1.5×10^{-88}

Verification 5.3: Applying both generators of $SL(2, \mathbb{Z})$ at $\tau = 0.31 + 1.27i$.

j		difference from $j(\tau)$
$j(\tau)$	$-357.8718154056521 - 2654.846593778724i$	—
$j(\tau + 1)$	the same	2.0×10^{-27}
$j(-1/\tau)$	the same	1.8×10^{-27}

(QED)

Observation 5.4

j is a function of the torus, not of the lattice one happened to draw.

Verification 5.5: Tori that are homeomorphic and not conformally equivalent.

τ	$j(\tau)$
i	1728.0
$2i$	287496.0
$3i$	153553679.39673
$0.5 + i$	−89.594997669383

(QED)

Observation 5.6

All four are the same topological surface. Their j values differ, so no two of the first three are conformally equivalent. Uniformization does not end in one shape — it ends in one model together with a point of a moduli space.

6. The sign of χ decides only for closed surfaces

Verification 6.1: By Gauss–Bonnet $\int K = 2\pi\chi$, so on a closed surface the sign of χ picks the model.

genus	χ	sign	model
0	2	+	sphere
1	0	0	plane (flat)
2	-2	-	disk (hyperbolic)
3	-4	-	disk (hyperbolic)
4	-6	-	disk (hyperbolic)

Verification 6.2: On an open surface it decides nothing.

surface	χ	homotopy type	model
the plane	1	a point	plane
the open disk	1	a point	disk

(QED)

Observation 6.3

Same χ , same homotopy type, different model. Which of the two an open surface is — the type problem — is settled by no topological invariant. The trichotomy of the universal cover and the trichotomy by the sign of χ agree on closed surfaces and part company on open ones — the same discipline by which Papers 67 and 128 fenced in where $4\pi = 2\pi\chi$ actually holds.

7. The “uni” is uniqueness after three real conditions

Fact 7.1: Every automorphism of the disk is $e^{i\theta} \frac{z-a}{1-\bar{a}z}$, with two real parameters in a and one in θ : **three real parameters**.

Verification 7.2: Reading a composition of two of them **off its values alone** recovers

$a = -0.4157923416 + 0.4781102072i$ and $\theta = -0.5242149765$, disagreeing at **15** sample points by 2.9×10^{-31} . Taking $a = z_0, \theta = 0$ at $z_0 = 0.35 - 0.2i$ gives $f(z_0) = 0$ and $f'(z_0) = 1.194029851$ with imaginary part 2×10^{-69} , that is **real and positive**. (QED)

Observation 7.3

The “uni” of uniformization is uniqueness after normalisation. Before it, a three-parameter family; after it, exactly one map. There is no “the map” before the conditions are imposed.

8. The census

Observation 8.1

object	address	separator	note
the three models	sphere / plane / disk	sign of curvature	topology does not split them
plane and disk	homeomorphic, not conformal	Liouville	bounded entire means constant
an annulus	one modulus	$\log(R/r)$ or $\operatorname{arccosh}$	topology does not see it
a torus	the j -invariant	τ modulo $SL(2, \mathbb{Z})$	model plus parameter
closed surfaces	sign of χ	genus	Gauss–Bonnet
open surfaces	χ decides nothing	the type problem	no topological invariant
uniqueness of the map	three real conditions	normalisation	before it, a family

Observation 8.2

“It splits into three” is not a sentence of topology. And “it is determined uniquely” is not right as it stands — what is determined is the model, and the parameter remains after it.

9. Conclusion

1. One right angle survives even a homeomorphism (Ver. 2.2, 2.4): distortion $1.25/2.0/5.0$ against 1.0.
2. The modulus agreed to 20 digits along two independent routes (Ver. 3.2).
3. A family of visibly different regions with one modulus was solved (Ver. 4.1): $r = 0.296707, \dots, 0.248139$.

4. j is invariant under $SL(2, \mathbb{Z})$ (Ver. 5.3): $j(i) = 1728, j(2i) = 287496$.
5. χ decides nothing on open surfaces (Ver. 6.2): plane and disk both have $\chi = 1$.
6. The “unique” is unique after three conditions (Ver. 7.2).

Closing: Uniformization is usually told as the theorem that folds every shape into one. It folds as far as the model, and after that one real number, or one complex number, is left over. Whether that leftover parameter is read as a remainder or as part of the answer is what changes how this theorem is read.

References

1. Riemann, B. (1851). *Grundlagen für eine allgemeine Theorie der Functionen einer veränderlichen complexen Grösse*. Göttingen: Dissertation.
 2. Koebe, P. (1907). *Über die Uniformisierung beliebiger analytischer Kurven*. Göttinger Nachrichten.
 3. Poincaré, H. (1907). *Sur l'uniformisation des fonctions analytiques*. Acta Mathematica, 31.
 4. Liouville, J. (1844). *Leçons sur les fonctions doublement périodiques*. Journal für die reine und angewandte Mathematik.
 5. Ahlfors, L. V. (1978). *Complex Analysis* (3rd ed.). New York: McGraw-Hill.
 6. Klein, F. (1878). *Über die Transformation der elliptischen Functionen und die Auflösung der Gleichungen fünften Grades*. Mathematische Annalen, 14.
 7. Yamagishi, Y. (2026). *The Euler-Characteristic Origin of the 4π Patch* [Paper 67]. Zenodo. 10.5281/zenodo.21642170
 8. Yamagishi, Y. (2026). *Curvature Is Intrinsic* [Paper 91]. Zenodo. 10.5281/zenodo.21758406
 9. Yamagishi, Y. (2026). *Hearing Shape from Spectrum* [Paper 102]. Zenodo. 10.5281/zenodo.21836735
 10. Yamagishi, Y. (2026). *Flowing Geometry to Canonical Form* [Paper 103]. Zenodo. 10.5281/zenodo.21836801
 11. Yamagishi, Y. (2026). *The Exception Only in Four Dimensions* [Paper 105]. Zenodo. 10.5281/zenodo.21836925
 12. Yamagishi, Y. (2026). *How Far Does Light Determine Geometry?* [Paper 111]. Zenodo. 10.5281/zenodo.21961685
 13. Yamagishi, Y. (2026). *$4\pi = 2\pi\chi$ Holds in Three Dimensions and Nowhere Else* [Paper 128]. Zenodo. 10.5281/zenodo.22079576
 14. Yamagishi, Y. (2026). *The Same Character Table, and Still Not Isomorphic* [Paper 152]. Zenodo. 10.5281/zenodo.22084202
 15. Yamagishi, Y. (2026). *The Definition Divides d by p ; the Theorem Divides p by the Discriminant* [Paper 166]. Zenodo. 10.5281/zenodo.22093052
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