

Sciama's Famous Coincidence Is an Identity

— A Census of Mach's Principle: $GM/(Rc^2)$ Is Exactly 1/2 for Every Hubble Constant — [Paper 169]

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— *From thinking to syncing.* —

ABSTRACT

The single phrase “Mach's principle” names several claims that do not imply one another. This paper calls them by name and shows in numbers that a much-quoted numerical coincidence is in fact an identity following from the definitions. **NO NEW PHYSICAL LAW IS CLAIMED** — nothing here contradicts general relativity. First, and this is the core. Computing $GM/(Rc^2)$ from the Hubble radius $R = c/H$ at the critical density gives exactly 0.5 at $H = 50, 67.4, 70, 100$ and 1000 — to every digit computed. Substituting $M = (4/3)\pi R^3 \rho_c = c^3/(2GH)$ cancels the H , so this is not a coincidence but an identity (Section 2). Second, away from the critical density the answer is simply $\Omega/2$ (0.05, 0.15, 0.5, 1.5 at $\Omega = 0.1, 0.3, 1, 3$). So what the “coincidence” records is $\Omega = 1$, that is flatness, and it says nothing whatever about the origin of inertia (Section 3). Third, geodetic precession and frame dragging have distinct roots. Around the Earth at $r = 7027$ km they are 6.6048684 against 0.081436822 arcseconds per year, a ratio of 81.1 — but at $r = 42164$ km the ratio is 198.7. Having different dependence on r , they are not one effect at two strengths (Section 4). Fourth, dragging is always partial. Thirring's weak-field coefficient $f = 4GM/(3Rc^2)$ is 9.28×10^{-10} for the Earth, 2.83×10^{-6} for the Sun and 0.2297 for a neutron star. Extrapolating to $f = 1$ gives R/r_s exactly 2/3 for every mass — inside the horizon (Section 5). Fifth, there is a counterexample from the other side. Minkowski space has inertial frames and no matter at all. “Inertia is produced entirely by distant matter” is false inside general relativity (Section 6).

Scope of this paper: **NO NEW PHYSICAL LAW IS CLAIMED** — nothing here contradicts general relativity. That Mach's principle is not one sentence, the Lense–Thirring effect, Thirring's shell coefficient, Gödel's universe and Sciama's argument are all standard. Nor does this paper rule on whether Mach's principle is right — what can be written is what each reading demands. What is added is computing $GM/(Rc^2)$ across values of H and confirming that it is an identity, showing that the ratio of geodetic precession to frame dragging moves with r , and showing that extrapolating the weak-field coefficient gives $R/r_s = 2/3$ for every mass, which proves the extrapolation invalid.

1. Notation and position

Write H for the Hubble constant, $R = c/H$ for the Hubble radius, $\rho_c = 3H^2/(8\pi G)$ for the critical density and $\Omega = \rho/\rho_c$ for the density parameter, and $r_s = 2GM/c^2$ for the Schwarzschild radius.

Position: Paper 97 traced the lineage of relationalism from Leibniz through Mach and general relativity to Rovelli but did not treat Mach's principle itself. This paper opens that one sentence and joins it to the separation of convention from fact in Paper 95 and to the inverse-problem family of Papers 89 and 159.

2. The core: $GM/(Rc^2)$ is an identity

Observation 2.1

The claim often quoted runs thus — **summing $Gm/(rc^2)$ over the visible universe gives something near 1, as if distant matter were paying for inertia.**

Verification 2.2: Compute the number from the Hubble radius at the critical density.

H (km/s/Mpc)	$R = c/H$ (m)	M inside (kg)	$GM/(Rc^2)$
50	1.85013×10^{26}	1.24568×10^{53}	0.5
67.4	1.3725×10^{26}	9.24096×10^{52}	0.5
70	1.32152×10^{26}	8.89772×10^{52}	0.5
100	9.25063×10^{25}	6.22841×10^{52}	0.5
1000	9.25063×10^{24}	6.22841×10^{51}	0.5

It is 0.5 to every digit computed. (QED)

Observation 2.3

The reason is visible on substitution alone.

$$M = \frac{4}{3}\pi\left(\frac{c}{H}\right)^3 \cdot \frac{3H^2}{8\pi G} = \frac{c^3}{2GH}, \qquad \frac{GM}{Rc^2} = \frac{G}{c^2} \cdot \frac{c^3}{2GH} \cdot \frac{H}{c} = \frac{1}{2}.$$

The H cancels. This is not a number that was measured and found to agree; it is an identity that follows from the definitions (in the six roots of the equals sign of Paper 16, a consequence of substitution, not an empirical law).

3. So what does the coincidence record?

Verification 3.1: Compute away from the critical density.

Ω	$GM/(Rc^2)$
0.1	0.05
0.3	0.15
1	0.5
3	1.5

The answer is simply $\Omega/2$. (QED)

Observation 3.2

So what the observation “it comes out near 1” records is $\Omega \simeq 1$, that is flatness. That is a fact about the geometry of the universe and says nothing about where inertia comes from.

Observation 3.3

What must be moved for the number to move is Ω — not the amount of matter, not the age of the universe, and not H . Multiplying H by twenty did not move it. A quantity that does not move cannot be offered as evidence.

4. Geodetic precession and dragging have distinct roots

Verification 4.1: For a gyroscope orbiting the Earth ($J = 0.3307MR^2\omega = 5.84571 \times 10^{33}$ kg m²/s), compute the two characteristic rates

geodetic : $\frac{3}{2} \frac{GM}{c^2 r} \sqrt{\frac{GM}{r^3}}$, dragging : $\frac{GJ}{c^2 r^3}$.

r (km)	geodetic (as/yr)	dragging (as/yr)	ratio
7027	6.6048684	0.081436822	81.1
10000	2.7339405	0.028257301	96.8
26561	0.23778147	0.0015079863	157.7
42164	0.074891847	0.00037696867	198.7

(QED)

Observation 4.2

The ratio moves from 81.1 to 198.7, because the geodetic rate goes as $r^{-5/2}$ and the dragging rate as r^{-3} .
With different dependence on r , these are not one effect at two strengths.

Verification 4.3: Stop the Earth's rotation and recompute.

geodetic ($J = 0, r = 7027$ km)	6.6048684 as/yr
dragging ($J = 0$)	0

(QED)

Observation 4.4

J does not appear in the geodetic rate at all — it survives unchanged around a body that does not rotate. The dragging rate is proportional to *J* and vanishes with it. One experiment holds two distinct roots, mass curvature on one side and rotation on the other (the same discipline by which Paper 165 separated the three equivalence principles).

Observation 4.5

And the Machian half is the small half.

5. Dragging is always partial

Verification 5.1: Thirring's weak-field result for the dragging fraction inside a slowly rotating thin shell is $f = 4GM/(3Rc^2)$.

shell	<i>M</i> (kg)	<i>R</i> (m)	<i>f</i>
the Earth	5.9722×10^{24}	6.371×10^6	9.28175×10^{-10}
the Sun	1.98847×10^{30}	6.957×10^8	2.83009×10^{-6}
a neutron star (1.4 solar masses, 12 km)	2.78386×10^{30}	1.2×10^4	0.229704

(QED)

Verification 5.2: Machian talk wants $f = 1$, complete dragging. Solving gives $R = 4GM/(3c^2)$; compare with $r_s = 2GM/c^2$.

<i>R</i> giving $f = 1$ (at the Earth's mass)	0.0059134019 m
Schwarzschild radius r_s	0.0088701029 m
ratio R/r_s	0.666666666667

The ratio is exactly 2/3, and it is 2/3 for every mass, since *M* cancels. (QED)

Observation 5.3

The weak-field formula reports that complete dragging is reached inside the horizon. That is not a discovery about black holes. It is a proof that the formula was carried outside its domain.

Observation 5.4

So whether complete dragging occurs is not settled by extrapolating a weak-field coefficient — it requires the exact treatment. This paper claims no such answer (as in Paper 159, where “ill posed” was separated, what is written is how far one's own tools reach).

6. The counterexample from the other side: vacuum has inertia

Fact 6.1

solution	matter	inertial frames
Minkowski space	none	well defined
Schwarzschild exterior	none locally	well defined
Gödel's universe (1949)	rotating dust	no global rest frame
Kerr exterior	none locally	dragged, still defined

Observation 6.2

General relativity has vacuum solutions carrying a perfectly good inertial structure. So the strong reading, that inertia is produced entirely by distant matter, is false inside the very theory usually credited with vindicating it.

Observation 6.3

The reason is structural. The field equations relate matter to curvature, and picking out one solution needs boundary conditions as well. Asymptotic flatness is a choice, not a consequence of the matter.

Observation 6.4

Read as an inverse problem, Mach asks whether the local inertial frame determines the distribution of matter. It does not — the same matter admits different boundary data. Where Paper 89 assembled tools for reading a hidden cause from a perfect null, here stands an example on the unreadable side.

7. The census

Observation 7.1

reading	claim	general relativity's answer	what moves it
dynamical universe	the universe is not static	satisfied	solutions are dynamical
the strong reading	an isolated body has no inertia	not satisfied	vacuum solutions
the local reading	matter fixes local inertial frames	partly	boundary conditions
the cosmological reading	$\Omega = 1$	a question for observation	Ω (Sections 2, 3)
dragging	rotation drags inertial frames	satisfied	J (and it is small)

Observation 7.2

These five are not a staircase of strength. General relativity satisfies some, violates others, and for others demands boundary conditions. The sentence “general relativity realised Mach's principle” has no truth value until it says which reading.

8. Conclusion

1. $GM/(Rc^2)$ is an identity (Ver. 2.2): multiplying H by twenty leaves 0.5.
2. What it records is $\Omega/2$, that is flatness (Ver. 3.1).
3. Geodetic and dragging rates depend differently on r (Ver. 4.1): the ratio runs from 81.1 to 198.7.
4. At $J = 0$ the geodetic rate survives and the dragging vanishes (Ver. 4.3).

5. Extrapolating the weak-field coefficient gives $R/r_s = 2/3$ (Ver. 5.2): inside the horizon, so the extrapolation is invalid.

6. Vacuum has inertia (Fact 6.1).

Closing: When a number agrees exactly, the first thing to suspect is not nature but the definitions. $GM/(Rc^2)$ was $1/2$ before anyone measured it. Mach's question does not vanish with it, but this number is no longer evidence for it — only a quantity that can be moved can serve as evidence.

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