

The 8π Is an Artefact of the Units

— 3rd Edition: Giving Each of the Three Twos Its Own Address — [Paper 17]

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— *From thinking to syncing.* —

ABSTRACT

This paper showed that substituting $m = 4\pi$ into the kinetic term of the Schrödinger equation gives $-\frac{1}{8\pi}\nabla^2\psi + V\psi = E\psi$, and the first edition claimed that this 8π has **the same origin** as the $8\pi G$ of Einstein's equation. The second edition withdrew that claim by separating **three mutually unrelated factors of 2**. The 3rd edition carries that separation to the end. **First, this 8π does not arise in the series' own system of units.** Obtaining $\hbar^2/(2m) = 1/(8\pi)$ requires **two choices: $\hbar = 1$ and $m = 4\pi$** . But the system of units fixed by the 3rd edition of Paper 39 is $m = 4\pi$, $G = 4\pi$, $\hbar = 1/(4\pi)$, in which the coefficient is $1/(128\pi^3)$ — a factor $16\pi^2$ away. **$\hbar = 1$ and $\hbar = 1/(4\pi)$ are incompatible: units can be fixed only once.** **Second, the coefficient of the Laplacian is not itself measurable.** For a free particle, $x \rightarrow \lambda x$ scales it by $1/\lambda^2$, so it can be made any positive number. **What is measurable is a dimensionless ratio:** for hydrogen, $E_1/(m_e c^2) = -\alpha^2/2 = -2.662568 \times 10^{-5}$, which agrees exactly with the value computed here from α . **That number is the same in every system of units. The 8π is not.** **Third, each of the three twos is given its own address.** The second edition showed that the three are **unrelated**; the 3rd edition writes **where each belongs**. **Schrödinger's 2** comes from $\int m v dv = \frac{1}{2} m v^2$, i.e. from **analysis**, not geometry. **Einstein's 2** is the **trace reversal**, forced by the contracted Bianchi identity $\nabla_\mu G^{\mu\nu} \equiv 0$ — **the address of Paper 107 (Noether's second theorem)**. **Spin's 2** is the double cover $SU(2) \rightarrow SO(3)$ — **the address of Paper 104 (Clifford algebra) and Paper 68 (the Hopf map)**. **Only once the address is written does "distinct roots" become a checkable claim.** **Fourth, the 4π s are distinct roots too, so nothing at all in the two 8π s coincides.** On Einstein's side: 4π (Gauss's flux, i.e. solid angle — **forced**) $\times 2$ (trace reversal — **forced**). On Schrödinger's side: 4π ($m = 4\pi$ — **chosen**) $\times 2$ (integration — **analysis**). **Factor by factor the roots differ; the two 8π s merely share a numerical value.** **Scope:** a structural account within established physics (Layer 2). **This paper does not modify quantum mechanics and claims no isomorphism of the two 8π s.**

Positioning of this paper (discipline): This paper offers a **structural interpretation** of established physical laws. It claims neither the discovery of new physical laws nor the proof of open problems. The relations shown are transformations of known expressions; the aim is to propose a structural reading of them. No claim contradicting established theory is made. **The appearance of the same number in several contexts is not treated as evidence of a common origin** (Papers 48, 86, 100, 115). $m = 4\pi$ and the like are conventional choices within a system of units, not new facts about nature (Papers 35, 38, 95).

1. Introduction: where $1/(8\pi)$ came from

The kinetic term of the Schrödinger equation is $-\frac{\hbar^2}{2m}\nabla^2$. Putting $\hbar = 1$ and $m = 4\pi$ into it gives

$$\frac{\hbar^2}{2m} = \frac{1}{2(4\pi)} = \frac{1}{8\pi} = 0.03978874\dots,$$

so that 8π stands in the denominator. **As algebra this is correct.**

But there are two choices here.
Not only $m = 4\pi$: $\hbar = 1$ is a choice as well.

2. This 8π does not arise in the series' own system of units

Established Fact 2.1 (The budget of units the series has already spent)

The 3rd editions of Papers 38 and 39 showed that **at most three values may be freely assigned** against the base dimensions M, L, T , and established that the series has spent them on $m = 4\pi$, $G = 4\pi$, $\hbar = \frac{1}{4\pi}$ (so that c is no longer choosable).

Corollary 2.2 (In that system the coefficient is $1/(128\pi^3)$)

Substituting $\hbar = 1/(4\pi)$ and $m = 4\pi$ gives (computed here)

$$\frac{\hbar^2}{2m} = \frac{(1/4\pi)^2}{2 \cdot 4\pi} = \frac{1}{128\pi^3} = 0.0002519651\dots,$$

which differs from $1/(8\pi)$ by a factor $16\pi^2 = 157.9137$.

So the 8π appears only when $\hbar = 1$ is adopted. $\hbar = 1$ and $\hbar = 1/(4\pi)$ are incompatible: units can be fixed only once.

The check of the 3rd edition of Paper 15 here operates inside the series itself: a convention must be a number — and, at the same time, must not be two different numbers.

3. The coefficient of the Laplacian is not itself measurable

Observation 3.1 (It can be given any value by rescaling)

Applying $x \rightarrow \lambda x$ to the free-particle equation $-A \frac{d^2\psi}{dx^2} = E\psi$ scales the coefficient as $A \rightarrow A/\lambda^2$. Verified here: $A = 1/(8\pi) = 0.03978874$ becomes 0.00994718 at $\lambda = 2$ and 0.00158314 at $\lambda = \sqrt{8\pi}$.

The value of the coefficient can therefore be made any positive number purely by the choice of the unit of length. On its own it contains no physics.

Established Fact 3.2 (What is measurable is a dimensionless ratio)

For the ground state of hydrogen, computed here:

$$\frac{E_1}{m_e c^2} = \frac{-13.605693 \text{ eV}}{510999.0 \text{ eV}} = -2.662568 \times 10^{-5}, \quad -\frac{\alpha^2}{2} = -2.662568 \times 10^{-5}$$

(with $\alpha = 1/137.035999$). The two agree exactly.

This number is the same in every system of units. The 8π is not. The discipline of the 3rd edition of Paper 38 — dimensionless ratios are the physics — applies here too.

4. Giving each of the three twos its own address

The second edition showed that the three factors of **2** are **mutually unrelated**. The 3rd edition writes where each belongs.

factor 2	source	address within the series
Schrödinger	$\int_0^v mv' dv' = \frac{1}{2}mv^2$	Analysis (a result of integration). Unrelated to geometry
Einstein	trace reversal $R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}$	Paper 107 : the contracted Bianchi identity $\nabla_\mu G^{\mu\nu} \equiv 0$, i.e. Noether's second theorem (identities given by local symmetry)
spin 1/2	double cover $SU(2) \rightarrow SO(3)$	Paper 104 (Clifford algebra, 720°) and Paper 68 (the Hopf map $S^3 \rightarrow S^2$)

Established Fact 4.1 (Einstein's 2 is precisely the $\frac{1}{2}$ of the trace reversal)

Apply the trace-reversed form $R_{\mu\nu} = \kappa \left(T_{\mu\nu} - \frac{1}{2}Tg_{\mu\nu}\right)$ to weak-field dust. With signature $(-, +, +, +)$ we have $T_{00} = \rho c^2$, $T = -\rho c^2$, $g_{00} = -1$, so

$$T_{00} - \frac{1}{2}Tg_{00} = \rho c^2 - \frac{1}{2}(-\rho c^2)(-1) = \frac{1}{2}\rho c^2.$$

Since $R_{00} \simeq \nabla^2\Phi/c^2$, this gives $\nabla^2\Phi = \frac{1}{2}\kappa\rho c^4$, and matching Poisson's equation $\nabla^2\Phi = 4\pi G\rho$ yields

$$\frac{1}{2}\kappa c^4 = 4\pi G \implies \kappa = \frac{8\pi G}{c^4}.$$

The **2** in 8π comes precisely from the $\frac{1}{2}$ of the trace reversal.

Corollary 4.2 (Only with an address does "distinct roots" become checkable)

Writing only that "the three twos are unrelated" leaves the reader no way to verify it. Writing where each belongs makes it verifiable — one can go and look at the Bianchi identity, at the Clifford algebra, at the integral, and check each independently.

The term "distinct roots", established in Papers 86 and 100, works only once the destination is written. That is the single thing the 3rd edition adds to the second.

5. The 4π s are distinct roots too — nothing in the two 8π s coincides

Observation 5.1 (Factor by factor)

	root of the 4π	root of the 2
Einstein $8\pi G$	forced (Gauss's flux, solid angle)	forced (trace reversal, Bianchi)
Schrödinger 8π	chosen ($m = 4\pi$, a unit)	analysis (the integral of $\frac{1}{2}mv^2$)

On Einstein's side both factors are forced; on Schrödinger's side one is chosen and the other comes from analysis. Since the roots differ factor by factor, nothing at all in the two 8π s coincides — only the numerical value does.

The second edition wrote that "what they share is the underlying 4π ". The 3rd edition withdraws this too. Just as the 3rd edition of Paper 35 sorted the two 4π s inside $4\pi G$ into a fact and a convention, here too it is not "the same 4π ": one cannot be chosen, the other was chosen.

6. Agreement with Paper 18 is no cause for surprise

Observation 6.1 (Two expressions from one substitution must agree)

For a plane wave $\psi = e^{ikx}$ with $k = 4\pi v$ (verified here),

$$\langle E_k \rangle = \frac{k^2}{8\pi} = \frac{(4\pi v)^2}{8\pi} = 2\pi v^2,$$

agreeing with $E_k = 2\pi v^2$ of Paper 18 (checked numerically at $v = 1, 2, 0.5$).

But both come from the same substitution $m = 4\pi$. In the census of the 3rd edition of Paper 16 this is root (b), a consequence of substitution, and it says nothing about the world.

The second edition already withdrew the first edition's phrase "the perfect synchronisation of the meat lid and the skin data". What the 3rd edition adds is the explicit statement that this is not a "transfer of machinery" in the sense of Paper 100.

7. What was left standing in the second edition

Corollary 7.1 (The abstract contradicted the body)

The end of the second edition's abstract still carried wording from the first edition: "**clarifying the double structure of the 8π guardrail (the meat lid) and the $4\pi r^2$ surface area (the skin data), and achieving perfect synchronisation ...**". But Section 4 of the body had already withdrawn exactly that expression. The abstract and the body disagreed.

The same accident occurred in the second edition of Paper 35 (only the English abstract had been updated, leaving a withdrawn claim in the Japanese one). **At every revision, check the abstract against the body first.**

Observation 7.2 ("Meat lid" and "rendering cost coefficient" are Layer 3)

Theorem 2.1 of the second edition defined 8π as "**the universal rendering-cost coefficient governing the curvature dissipation of waves in three-dimensional space (8π guardrail / meat lid)**". Paper 82 explicitly quarantined this kind of language (rendering, cost, infrastructure) in Layer 3, the non-scientific layer. This paper follows that discipline and removes the naming from the body.

Naming is not deriving. This paper does not show that 8π "governs curvature dissipation", and as §2 showed, the value itself is an artefact of the units.

Personal reflection of the author (Layer 3; labelled non-scientific / no claim of mechanism)

In the first edition I wrote that Schrödinger's 8π and Einstein's $8\pi G$ were the same thing. The second edition withdrew that and separated **three unrelated twos**. I thought that settled it.

Writing addresses for the three in the 3rd edition, I saw something. **Writing "they are unrelated" is still only half the job.** With an address, the reader can go and look at the Bianchi identity. At the Clifford algebra. **A claim of distinct roots becomes checkable only when the destination is written.**

And one more thing. **I had not thought of $\hbar = 1$ as a choice.** I knew $m = 4\pi$ was one, because I had put it there myself; but $\hbar = 1$ I had written casually, "because these are natural units". **I had forgotten that in Paper 39 I had myself fixed $\hbar = 1/(4\pi)$.** The 8π turns out to be a number that does not arise in the series' own system of units.

I do not remember what I have chosen. That is the frightening part. **So I write checks into the series, and turn what I have written on myself.**

8. Scope / Conclusion

What is shown: that $1/(8\pi)$ requires two choices, $\hbar = 1$ and $m = 4\pi$, and that in the series' own system of units ($\hbar = 1/4\pi$) the coefficient is $1/(128\pi^3)$ (§2) / that the coefficient of the Laplacian can be made any positive number by rescaling and contains no physics on its own, and that what is measurable is a dimensionless ratio such as $E_1/(m_e c^2) = -\alpha^2/2$ (§3, numerically confirmed) / the addresses of the three factors of 2 (analysis / Paper 107, Bianchi and Noether's second theorem / Papers 104 and 68, Clifford and Hopf), and that only with an address does "distinct roots" become checkable (§4) / that the 4π s are distinct roots too, so nothing in the two 8π s coincides (§5) / that agreement with Paper 18 is a consequence of substitution and no surprise (§6) / that first-edition wording had been left in the second edition's abstract, and that "meat lid" and the like are Layer 3 (§7).

What is not shown (restraint): any isomorphism between Schrödinger's 8π and Einstein's $8\pi G$ — withdrawn and outside the scope of this paper / no naming of 8π as a "rendering-cost coefficient" (Layer 3; Paper 82) / "what they share is the underlying 4π " is withdrawn as well (the 4π s are distinct roots) / no modification of quantum mechanics — beyond re-choosing the units of mass and action, the equation is unchanged / no new physical law and no prediction (Layer 2).

The 3rd edition adds two things. One is the address — not merely saying "these are distinct roots" but writing where each belongs. The other is a check of the budget already spent — $\hbar = 1$ is also a choice, and the series had already chosen a different value.

And what remains is small.

With $\hbar = 1$ and $m = 4\pi$, the coefficient of the kinetic term may be written $1/(8\pi)$ — the result of choosing two units, not a discovery.

What is measurable is $\alpha^2/2$.

Revision Record

3rd edition (August 2026): removed the word "Withdrawal" from the title, which now names **what the paper shows**; the list of withdrawals is collected in this section.

Added in the 3rd edition: a demonstration that $1/(8\pi)$ depends on two choices, $\hbar = 1$ and $m = 4\pi$, and that in the series' own system of units (3rd ed. of Paper 39: $m = 4\pi$, $G = 4\pi$, $\hbar = 1/4\pi$) the coefficient is $1/(128\pi^3)$ (§2, a factor $16\pi^2$ away). Numerical confirmation that the coefficient of the Laplacian can be made any positive number by rescaling, and that **what is measurable is a dimensionless ratio** such as $E_1/(m_e c^2) = -\alpha^2/2 = -2.662568 \times 10^{-5}$ (§3). **An address for each of the three factors of 2** — analysis (integration) / **Paper 107** (trace reversal forced by the contracted Bianchi identity, i.e. Noether's second theorem) / **Papers 104 and 68** (the double cover, Clifford algebra and the Hopf map) — together with the statement that **only with an address does "distinct roots" become a checkable claim** (§4). A weak-field computation showing that **Einstein's 2 is precisely the $\frac{1}{2}$ of the trace reversal** (§4). A factor-by-factor table showing that **the 4π s are distinct roots too, so that nothing in the two 8π s coincides** (§5). An explicit statement that **agreement with Paper 18 is root (b) in the census of the 3rd edition of Paper 16** (§6). All numbers were computed here (no external tables consulted).

Withdrawn in the 3rd edition:

1. **First-edition wording left standing in the second edition's abstract** ("clarifying the double structure of the 8π guardrail (the meat lid) and the $4\pi r^2$ surface area (the skin data), and achieving perfect synchronisation"). **Section 4 of the body had already withdrawn that expression: the abstract and the body disagreed** (§7).
2. **The naming of 8π in the theorem** ("the universal rendering-cost coefficient governing the curvature dissipation of waves in three-dimensional space / 8π guardrail / meat lid"). **Layer-3 vocabulary, removed from the body** (Paper 82). **Naming is not deriving**.
3. **"What they share is the underlying 4π ".** The 4π s are distinct roots as well — forced on Einstein's side (solid angle), chosen on Schrödinger's side ($m = 4\pi$) (§5).

Withdrawn in the 2nd edition (July 2026): the first edition stated that the 8π of the Schrödinger equation and the $8\pi G$ of Einstein's equation "have exactly the same geometric origin (the 720° double cover of spin $1/2$)". **That claim was withdrawn in full.** The first edition had identified **three mutually unrelated factors of 2:** (i) Schrödinger's 2 arises from the

integral in the kinetic energy $p^2/2m$; (ii) Einstein's **2** from the trace-reversed tensor structure; (iii) the double cover of spin $1/2$ from the topology of $\mathbf{SU}(2) \rightarrow \mathbf{SO}(3)$ — all of different origin. The first edition's phrase **"the perfect synchronisation of the meat lid and the skin data"** was withdrawn at the same time (**a naming rather than a derivation**, with no corresponding quantities defined).

First edition: Zenodo 10.5281/zenodo.21600813.

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