

Cancelling the Differentials Gives -1, Not +1

— Notation Is a Convention, and a Convention Is Not Harmless — [Paper 170]

Yuuki YAMAGISHI (山岸 勇輝)

Yamagishi Complexity Torus Lab, Japan

August 2026

— *From thinking to syncing.* —

ABSTRACT

A notation is a convention and moves no theorem. It does move what occurs to one next. This paper sets side by side, in one computation, the places where Leibniz's dy/dx keeps its promise and the places where it breaks it. **NO NEW MATHEMATICAL THEOREM IS CLAIMED** — all of this is standard. First, notation does not move the answer. Written with Newton's dot or with Leibniz's d , the derivative of $f(x) = \sin(x^2)e^{-x}$ at $x = 0.7$ is the same **0.37970904324369351073** (Section 2). Second, and this is the core. dy/dx looks like a fraction, and in the one-variable chain rule it really does behave like cancellation. But once three variables are tied by one relation, cancelling gives **-1** — for the ideal gas and at three points of the implicit surface $x^3 + y^3 + z^3 + xyz = 10$ alike, every value is **-1.0** (Section 3). Third, at second order the fraction reading collapses. With $x = t^3$ and $y = t^2$ the true d^2y/dx^2 is **-0.013888888888889** at $t = 2$ while the fraction reading gives **+0.166666666666667** — not the value but the sign is different (Section 4). Fourth, the same notation keeps one of its promises. The product rule really is the binomial theorem (exact through $n = 6$), with $n + 1$ terms of total weight 2^n . The chain rule does not — its terms number the partitions and their weights total the Bell numbers, **1024** against **115975** at $n = 10$ (Section 5). Fifth, one symbol plays two roles. With $z = x^2 + y^2$ and $y = 3x$, holding y fixed and letting y follow differ by a factor of ten — the subscript is not decoration (Section 6).

Scope of this paper: **NO NEW MATHEMATICAL THEOREM IS CLAIMED** — the triple product relation, the fact that d^2y/dx^2 is not a fraction, Leibniz's product rule, Faà di Bruno's formula with the Bell numbers, and the subscript on a partial derivative are all standard. Nor does this paper rule on the historical priority dispute — only the behaviour of the notation is treated. What is added is setting out, in one computation, how a single notation behaves at five different places, and in particular putting the product rule's weight 2^n and the chain rule's Bell numbers into one table.

1. Notation and position

Every derivative here was computed numerically. A **convention** is an agreement that can be chosen differently without moving any fact (Paper 95).

Position: Paper 95 separated convention from fact and wrote that “being a convention” does not mean “anything goes”. This paper gives one instance of that, for the thinnest convention of all — thin, and still deciding which mistakes are available.

2. Notation does not move the answer

Verification 2.1: The derivative of $f(x) = \sin(x^2)e^{-x}$ at $x = 0.7$.

computed numerically	0.37970904324369351073
from the closed form	0.37970904324369351073
difference	4.93×10^{-32}

(QED)

Observation 2.2

Newton would write $\dot{f}$ and Leibniz df/dx . The number is the same. A notation is a convention and moves no theorem. Everything below stands on that premise.

3. The core: cancelling gives -1

Observation 3.1

dy/dx looks like a fraction. In the one-variable chain rule $\frac{dy}{dt} = \frac{dy}{dx} \frac{dx}{dt}$ it really does behave like cancellation. That success is what gets carried into the next situation.

Verification 3.2: For the ideal gas $PV = nRT$ with $nR = 1$, compute the three partial derivatives, each holding the third variable fixed.

$\partial P/\partial V$ at fixed T	-0.75
$\partial V/\partial T$ at fixed P	0.6666666666666667
$\partial T/\partial P$ at fixed V	2.0
their product	-1.0

Verification 3.3: It is no quirk of the ideal gas. Repeat on the implicit surface $F = x^3 + y^3 + z^3 + xyz - 10 = 0$.

x	y	z	the triple product
1.0	1.0	1.833751	-1.0
1.5	0.5	1.732544	-1.0
0.7	1.3	1.7990588	-1.0

(QED)

Observation 3.4

Cancelling gives -1 every time. The notation invites the cancellation and the cancellation drops a sign.

Observation 3.5

Newton's dot cannot invite it — it has no denominator. The two notations write the same theorems and differ in which mistakes are available. What a convention selects is not truth but slipperiness.

4. At second order the fraction reading collapses

Verification 4.1: With $x = t^3$ and $y = t^2$, compare the true d^2y/dx^2 with the reading as a ratio of second derivatives.

t	the true d^2y/dx^2	read as a ratio
1.0	−0.22222222222222	+0.33333333333333
2.0	−0.013888888888889	+0.16666666666667
3.0	−0.0027434842249657	+0.11111111111111

The true value is $-2/(9t^4)$ and the ratio reading gives $+1/(3t)$. (QED)

Observation 4.2

Not merely a different value — a different sign. d^2y/dx^2 is one symbol, not a fraction of two things. Being right at first order guaranteed nothing at second.

5. One promise kept, one broken

Verification 5.1: The product rule. With $u = e^{2x}$, $v = \sin 3x$ at $x = 0.4$, compare the n -th derivative computed directly with $\sum_k \binom{n}{k} u^{(k)} v^{(n-k)}$.

n	directly	the binomial sum
1	6.567908305582929	6.567908305582929
2	−0.6941515037403181	−0.6941515037403181
4	−343.6136864015332	−343.6136864015332
6	3553.448468147446	3553.448468147446

They agree. (QED)

Verification 5.2: The chain rule. With $g = \sin$ and $h = e^x - 1$ at $x = 0.3$, apply Faà di Bruno's formula at $n = 4$.

$d^4g(h(x))$ directly	−15.82942435520884
Faà di Bruno, four grouped terms	−15.82942435520884
difference	0

The multiplicities used were **1; 4** and **3; 6; 1**, summing to **15 = B₄**. (QED)

Verification 5.3: Set the two growths side by side.

<i>n</i>	product: terms	product: weight	chain: terms	chain: weight
2	3	4	2	2
4	5	16	5	15
6	7	64	11	203
10	11	1024	42	115975

(QED)

Observation 5.4

In the product rule the notation's promise is kept exactly — the coefficients are the binomial coefficients and the weights total 2^n . In the chain rule it is not — the terms number the partitions and their weights total the Bell numbers, more than a hundredfold apart at $n = 10$.

Observation 5.5

One notation takes different attitudes to two neighbouring rules. So “ d may be used like a fraction” can only be settled rule by rule. What comes out of a notation is a conjecture, not a proof.

6. One symbol, two roles

Verification 6.1: Let $z = x^2 + y^2$ with $y = 3x$. Both of the following are called “the derivative of z with respect to x ”.

<i>x</i>	with <i>y</i> held fixed	with <i>y</i> following <i>x</i>	ratio
1.0	2.0	20.0	10.0
2.0	4.0	40.0	10.0
5.0	10.0	100.0	10.0

(QED)

Observation 6.2

A factor of ten between two things the bare notation writes the same way. That is why thermodynamics writes the held variable as a subscript — the subscript is not decoration, and without it the expression denotes no number at all (the same shape as what Paper 16 wrote about the equals sign).

7. How the notation was eventually justified

Observation 7.1

Using $d\mathbf{x}$ as an “infinitesimal” was for a long time an operation that worked without grounds. It was justified in Cartan's differential forms, where $d\mathbf{x}$ became a defined object, a 1-form, and “cancellation” became a defined operation, pullback along a map (Paper 92).

Observation 7.2

Leibniz's notation, in other words, was given its meaning two centuries later. It kept working throughout, and working is not the same as meaning something. Sections 3 to 5 are the limits of that working.

8. The census

Observation 8.1

place	appearance	fact	what moves it
one-variable chain rule	looks like cancelling	correct	nothing
three tied variables	cancelling gives +1	−1	the number of variables
d^2y/dx^2	looks like a fraction	even the sign differs	the order
the product rule	looks binomial	it is binomial	nothing
higher chain rule	looks simple	Bell numbers	the order
partial derivatives	one symbol	two meanings	what is held fixed

Observation 8.2

A notation does not move what is true. It moves which error looks natural. And that moves what occurs to one next.

9. Conclusion

1. Notation does not move the answer (Ver. 2.1).
2. Cancelling gives **−1** (Ver. 3.2, 3.3): **−1.0** for the gas and for the surface alike.
3. At second order even the sign differs (Ver. 4.1): **−0.0138889** against **+0.1666667**.
4. The product rule keeps the promise and the chain rule breaks it (Ver. 5.1–5.3): **$2^{10} = 1024$** against **$B_{10} = 115975$** .
5. The subscript is not decoration (Ver. 6.1): **a factor of ten**.

Closing: A convention may be chosen freely. What cannot be chosen is which mistake it invites. Leibniz's *d* has supplied correct guesses for three centuries and, over the same three centuries, has made people slip in the same places — and where they slip was settled the moment the notation was chosen.

References

1. Newton, I. (1687). *Philosophiae Naturalis Principia Mathematica*. London: Royal Society.
2. Leibniz, G. W. (1684). *Nova methodus pro maximis et minimis*. Acta Eruditorum.

3. Faà di Bruno, F. (1855). *Sullo sviluppo delle funzioni*. Annali di Scienze Matematiche e Fisiche, 6.
4. Bell, E. T. (1934). *Exponential polynomials*. Annals of Mathematics, 35(2).
5. Cartan, É. (1899). *Sur certaines expressions différentielles et le problème de Pfaff*. Annales scientifiques de l'École Normale Supérieure, 16.
6. Robinson, A. (1966). *Non-standard Analysis*. Amsterdam: North-Holland.
7. Yamagishi, Y. (2026). *The Equals Sign Has Distinct Roots Too* [Paper 16], 3rd ed. Zenodo. 10.5281/zenodo.21600471
8. Yamagishi, Y. (2026). *The Shared Alphabet* [Paper 92]. Zenodo. 10.5281/zenodo.21758486
9. Yamagishi, Y. (2026). *Convention and Fact* [Paper 95]. Zenodo. 10.5281/zenodo.21758621
10. Yamagishi, Y. (2026). *Time and Space as Relation* [Paper 97]. Zenodo. 10.5281/zenodo.21758697
11. Yamagishi, Y. (2026). *The **2** in Itô's Lemma Is Not a Dimension* [Paper 132]. Zenodo. 10.5281/zenodo.22080358
12. Yamagishi, Y. (2026). *$\epsilon^2 = 0$ Is a Definition of a Ring and $(dW)^2 = dt$ Is a Fact About a Path* [Paper 148]. Zenodo. 10.5281/zenodo.22083237
13. Yamagishi, Y. (2026). *The Definition Divides **d** by **p**; the Theorem Divides **p** by the Discriminant* [Paper 166]. Zenodo. 10.5281/zenodo.22093052
14. Yamagishi, Y. (2026). *What Lebesgue Bought Was Not a Bigger Class but Closure under Limits* [Paper 168]. Zenodo. 10.5281/zenodo.22093320

— From thinking to syncing. —

Yuuki YAMAGISHI (山岸 勇輝)

© 2026 Yamagishi Complexity Torus Lab, Japan · Licensed under Creative Commons Attribution 4.0 International (CC BY 4.0)