

Kodaira's Fence Is Integrality, Not Positivity

— Nash Assumes Nothing and Pays in Dimension; Kodaira Imposes a Condition and Gains Projective Space —

[Paper 173]

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— *From thinking to syncing.* —

ABSTRACT

Paper 158 treated Nash — every Riemannian manifold embeds isometrically, with no hypothesis at all. Kodaira's theorem has the opposite shape — an embedding into projective space exists exactly when a hypothesis holds, and that hypothesis is not vacuous. **NO NEW MATHEMATICAL THEOREM IS CLAIMED.** First, the convention was checked. On a product of elliptic curves the first Riemann relation vanishes and the Hermitian form has smallest eigenvalue 2.2 and 1.8 (Section 2). Second, and this is the core. For a randomly chosen complex 2-torus, an exhaustive search over the 15624 integral alternating forms with entries at most 2 found not one that works — the first relation stays at 0.0139392, 0.00445366 and 0.0273488 at best, never coming near zero (Section 3). Third, behind that lies a dimension count. All complex 2-tori form a real 8-dimensional family and the abelian surfaces a real 6-dimensional one — codimension 2. In complex dimension 1 the count reads 2 against 2, which is why the obstruction is invisible until dimension 2 (Section 4). Fourth, the fence is integrality. Scaling a working form by 2, 3, 0.5 or $\sqrt{2}$ leaves the first relation at 0 and the Hermitian form positive — all that breaks is integrality (Section 5). Fifth, Nash pays in dimension. His bound $n(3n + 11)/2$ gives 7, 17, 30, 46, 205 at $n = 1, 2, 3, 4, 10$ — the two theorems are not a staircase (Section 6).

Scope of this paper: **NO NEW MATHEMATICAL THEOREM IS CLAIMED** — Kodaira's embedding theorem, the Riemann relations, the fact that a general complex torus is not algebraic, and Nash's embedding theorem are all standard. The search in Section 3 is bounded — what is shown is that no integral form with entries up to 2 works, not that no polarisation exists. The general statement rests on the dimension count of Section 4, which is a standard fact. What is added is running the search over polarisations exhaustively and reporting the numbers, showing by rescaling that positivity survives and only integrality breaks, and setting Nash's and Kodaira's hypotheses, preserved structures and ambient spaces into one table.

1. Notation and position

Write a complex 2-torus as $\mathbb{C}^2/\Lambda$ with period matrix $\Omega = (I \mid Z)$, where Z is a 2×2 complex matrix. For an integral alternating form J on Λ (a 4×4 matrix with 6 independent entries), the **Riemann relations** are $\Omega J^{-1} \Omega^T = 0$ together with positive definiteness of $i \Omega J^{-1} \bar{\Omega}^T$. The sign is a matter of choosing J or $-J$ and is not part of the condition (the orientation making it positive is taken here).

Position: Paper 158 treated Nash's embedding and showed that the intrinsic and the extrinsic are dual and still have distinct roots. This paper adds the second voice — the embedding that demands a hypothesis. And the fence turns out to be the one Papers 146 and 147 described for quantisation — integrality.

2. Checking the convention

Verification 2.1: With Z diagonal, $\mathbb{C}^2/\Lambda$ is a product of elliptic curves and is **projective**. The principal polarisation should satisfy both relations.

Z	size of the first relation	smallest eigenvalue
diag(0.3 + 1.1 <i>i</i> , 1.7 <i>i</i>)	0.0	2.2
diag(2.0 <i>i</i> , 0.5 + 0.9 <i>i</i>)	0.0	1.8

(QED)

Verification 2.2: For the principal polarisation the condition is that Z be symmetric with $\text{Im } Z > 0$.

Z	size of the first relation	smallest eigenvalue
symmetric, $\text{Im } Z > 0$	0.0	1.3801961
not symmetric	0.583095	1.5168239

(QED)

Observation 2.3

What the first relation asks for is the symmetry of Z .

3. The core: a random torus admits no polarisation

Verification 3.1: Build every integral alternating form with entries from -2 to 2 , that is $5^6 - 1 = 15624$ of them, and count those satisfying both relations.

torus	search bound	number found	best first relation
symmetric Z (an abelian surface)	1	2	0.0
random Z , sample 1	2	0	0.0139392
random Z , sample 2	2	0	0.00445366
random Z , sample 3	2	0	0.0273488

(QED)

Observation 3.2

The abelian surface is found at once at $B = 1$. For random period matrices all 15624 forms fail, and the first relation never even comes near zero.

Observation 3.3

The search is bounded, however — what is shown is that nothing with entries up to 2 works, not that no polarisation exists. The general statement is carried by the count in the next section.

4. The dimension count

Fact 4.1

family	complex dimension	real dimension
all complex 2 -tori (Z arbitrary)	4	8
abelian surfaces (Z symmetric, $\text{Im } Z > 0$)	3	6
all complex 1 -tori	1	2
elliptic curves	1	2

Observation 4.2

Codimension 2. Allowing every integral polarisation rather than only the principal one replaces one such subset by a countable union of them, and the union is still of measure zero. Almost every complex torus of dimension 2 or more is not algebraic.

Observation 4.3

In complex dimension 1 the count reads **2** against **2** — every 1-torus is an elliptic curve, so the obstruction is invisible until complex dimension 2. “Invisible” and “absent” are different (the same shape as Paper 105 on what appears only in dimension four).

5. The fence is integrality

Verification 5.1: Rescale a working polarisation by a positive number.

scale	first relation	smallest eigenvalue	integral?
1	0.0	1.3801961	yes
2	0.0	0.69009805	yes
3	3.8×10^{-32}	0.46006537	yes
0.5	0.0	2.7603922	no
$\sqrt{2}$	7.8×10^{-32}	0.97594602	no

(QED)

Observation 5.2

At every scale the first relation stays at zero and the Hermitian form stays positive. The only thing that can break is whether the entries are integers. Positivity survives rescaling and integrality does not — so the fence is integrality.

Observation 5.3

A Kähler class can be positive and still fail to lie in the image of $H^2(X, \mathbb{Z})$, and then there is no embedding. This is the same fence as in Paper 146, where what quantises is that the gluing closes up, and Paper 147, where the same H^2 with different coefficients quantises nothing.

6. Nash and Kodaira

Fact 6.1

theorem	hypothesis	preserves	ambient
Nash 1956	none	the metric	$\mathbb{R}^N$, N growing
Kodaira 1954	a positive line bundle	the complex structure	one $\mathbb{P}^N$

Verification 6.2: The dimension Nash pays, $n(3n + 11)/2$.

n	N
1	7
2	17
3	30
4	46
10	205

(QED)

Observation 6.3

The two are not a staircase of strength. Nash demands nothing and pays with an ambient space that keeps growing. Kodaira demands a condition and needs only one kind of ambient space, where moreover the image is cut out by polynomials — so what “embedded” buys is far more.

Observation 6.4

They preserve different things too. Nash preserves the metric and Kodaira the complex structure — the metric only up to the Fubini–Study class. One word, “embedding”, naming two different things (one step beyond the distinction Paper 158 drew between intrinsic and extrinsic).

7. The census

Observation 7.1

item	content	what moves it	note
Nash	no hypothesis	the dimension N	preserves the metric
Kodaira	a positive line bundle	integrality	preserves the complex structure
the fence	integrality	not positivity	broken by rescaling
counterexample	a general complex torus	complex dimension ≥ 2	codimension 2, measure zero
where it hides	complex dimension 1	always algebraic	elliptic curves

Observation 7.2

The sentence “manifolds can be embedded” has no truth value until it says which structure the embedding preserves.

8. Conclusion

1. **The convention was checked (Ver. 2.1, 2.2): the first relation vanishes on a product of elliptic curves, and symmetry is what it asks for.**
2. **A random torus admits no polarisation (Ver. 3.1): all 15624 fail.**
3. **Codimension 2 (Fact 4.1): 8 against 6; in complex dimension 1, 2 against 2.**
4. **The fence is integrality (Ver. 5.1): positivity survives rescaling and integrality does not.**
5. **Nash pays in dimension (Ver. 6.2): 205 at $n = 10$.**

Closing: Nash's theorem says “it can be done” and Kodaira's says “it can be done when this holds”. The one with a condition is not the weaker — what the condition buys is a better ambient space. And the condition being paid is not size and not smoothness but being an integer.

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