

Whose Singularity Is It?

— Hironaka and Perelman: the Rhyme Is in the Method and Not in the Subject — [Paper 174]

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— *From thinking to syncing.* —

ABSTRACT

Hironaka's resolution of singularities and surgery in Ricci flow look much alike — each replaces a bad local model by a standard one, finitely often. This paper writes that likeness as a rhyme and puts three separators under it, in numbers. **NO STRUCTURAL IDENTITY IS CLAIMED** — refusing to claim one is the purpose of the paper. First, a blow-up is an isomorphism away from its centre. The map $(X, Y) \mapsto (X, XY)$ round-trips exactly for $X \neq 0$, and the whole line $X = 0$ collapses to one point — it is birational and has an inverse, the blow-down (Section 2). Second, resolution really terminates. The node and the cusp each take 1 blow-up, while $y^2 = x^5, x^7, x^9$ take 2, 3, 4 (multiplicity sequences $[2, 2, 1], [2, 2, 2, 1], [2, 2, 2, 2, 1]$) (Section 3). Third, the first separator is invertibility. A blow-up never touches the fundamental group (and adds 1 to χ); surgery is performed in order to change it (Section 4). Fourth, and this is the sharpest. A round 3-sphere has no singularity anywhere at $t = 0$, and its curvature blows up at $t = 0.25$ (250000 at $t = 0.249999$) — Hironaka removes a defect that was already there, Perelman one his own tool made (Section 5). Fifth, surgery needs a scale. Changing the threshold to 10, 100, 10^4 , 10^6 changes both the time of the cut and the radius of the neck — a blow-up has no such freedom (Section 6).

Scope of this paper: **NO NEW MATHEMATICAL THEOREM IS CLAIMED** — Hironaka's theorem, the local calculation of a blow-up, the round-sphere solution of Ricci flow and Perelman's surgery are all standard. And no structural identity is claimed — Paper 110 wrote that surgery and renormalisation are a rhyme, and this paper applies that discipline to a second, more tempting pair. A rhyme is a licence to look, never a ground to conclude. What is added is tracking multiplicity sequences with home-made polynomial arithmetic to count the blow-ups, showing in numbers that the curvature blow-up is absent at $t = 0$, and varying the threshold to show that where one cuts is a convention.

1. Notation and position

The **multiplicity** of a plane curve at the origin is the lowest total degree occurring. The **strict transform** is what remains after dividing out the exceptional factor. The polynomial arithmetic here was implemented over the rationals by hand.

Position: Paper 103 described Ricci flow as “dynamical uniqueness”, and Paper 110 set the identity of Ricci flow and renormalisation group flow in five languages and then said plainly that the likeness between surgery and renormalisation is a rhyme. This paper turns that discipline on a more tempting pair.

2. A blow-up is an isomorphism away from its centre

Verification 2.1: The map $(X, Y) \mapsto (X, XY)$ in the chart $y = XY$.

(X, Y)	image (x, y)	recovered (X, Y)
(1.0, 2.0)	(1.0, 2.0)	(1.0, 2.0)
(0.5, −3.0)	(0.5, −1.5)	(0.5, −3.0)
(2.0, 0.25)	(2.0, 0.5)	(2.0, 0.25)

For $X \neq 0$ the round trip is exact. (QED)

Observation 2.2

At $X = 0$ every Y maps to $(0, 0)$ — a whole line collapses to one point, and over that point stands a projective line. So a blow-up is an isomorphism outside a set of lower dimension — a birational map, with an inverse, the blow-down.

3. Resolution really terminates

Verification 3.1: Blow up until the strict transform is smooth at the origin.

curve	multiplicity sequence	blow-ups
node $y^2 = x^3 + x^2$	[2]	1
cusp $y^2 = x^3$	[2, 1]	1
$y^2 = x^5$	[2, 2, 1]	2
$y^2 = x^7$	[2, 2, 2, 1]	3
$y^2 = x^9$	[2, 2, 2, 2, 1]	4

(QED)

Observation 3.2

The multiplicity 2 comes down while spending two units of exponent at a time. The procedure terminates, and by Hironaka (1964) it terminates for every variety over a field of characteristic zero.

Observation 3.3

And the first fence already stands here — in characteristic p the theorem is open in dimension four and above. Ricci flow has no parameter that could even be set to p . The two “surgeries” do not share a parameter space.

4. First separator: one is invertible and the other is not

Observation 4.1

operation	inverse	information
blow-up	blow-down exists	kept (birational)
Ricci flow surgery	none	discarded

Observation 4.2

Their effects on the invariants run opposite too.

operation	π_1	b_2	χ
blow up a surface at a point	unchanged	+1	+1
surgery on a neck	changes	—	0 either way

Blowing up the projective plane at k points gives $\chi = 3 + k$, that is **3, 4, 5, 6, 7** for $k = 0, \dots, 4$. (QED)

Observation 4.3

A blow-up does not lay a finger on the fundamental group. Surgery is performed precisely in order to change it — splitting the manifold along its prime decomposition. One preserves the object up to birational equivalence and the other takes it apart. Their purposes point in opposite directions.

5. Second separator: whose singularity is it?

Verification 5.1: Under Ricci flow a round n -sphere of radius r obeys $r(t)^2 = r_0^2 - 2(n-1)t$.

n	r_0	extinction time	smooth at $t = 0$?
2	1.0	0.5	yes
3	1.0	0.25	yes
3	2.0	1.0	yes
4	1.0	0.166666666667	yes

Verification 5.2: The curvature scale ($n = 3$, $r_0 = 1$).

t	$1/(\tau_0^2 - 4t)$
0.0	1.0
0.1	1.666666667
0.2	5.0
0.2499	2500.0
0.249999	250000.0

(QED)

Observation 5.3

At $t = 0$ there is no singularity anywhere. The blow-up of the curvature was manufactured by the flow.

Observation 5.4

Hironaka removes a defect that was there before anyone arrived. Perelman removes one his own tool produced. One phrase, “resolving a singularity”, naming two things of opposite provenance — this is the sharpest separator.

6. Third separator: surgery needs a scale

Observation 6.1

A blow-up has no free parameter — the exceptional divisor is the projectivised tangent cone, and nothing is chosen. Surgery has one — a neck is cut where the curvature reaches a chosen threshold.

Verification 6.2 ($n = 3, r_0 = 1$)

threshold	time of the cut	radius at the cut
10	0.225	0.316227766
100	0.2475	0.1
10^4	0.249975	0.01
10^6	0.24999975	0.001

(QED)

Observation 6.3

Every threshold gives a different time and a different neck radius. In the language of Paper 95 this is a convention, and the theorem's real work is to show that the answer does not depend on it. Hironaka has no such clause to discharge.

7. So what is the rhyme worth?

Observation 7.1

It is worth something. Both replace a bad local model by a standard one, both do so finitely often before the object becomes usable, and both secure termination by a monotone quantity that drops at each step. That is a real pattern — but a pattern of method, not of subject.

Observation 7.2

A rhyme of method grants a licence to go and look. It grants no conclusion. The three separators above are a record of where that licence runs out.

8. The census

Observation 8.1

item	Hironaka (blow-up)	Perelman (surgery)
provenance of the singularity	already there	made by the flow
invertibility	blow-down exists	none
fundamental group	unchanged	changed on purpose
choice of scale	none	needed (a threshold)
parameter space	has a characteristic (p open)	has none
what they share	bad local model to standard form	finitely often; a monotone quantity

9. Conclusion

1. A blow-up is birational and has an inverse (Ver. 2.1).

2. Resolution terminates (Ver. 3.1): 1, 1, 2, 3, 4 blow-ups.

3. They part over invertibility and the fundamental group (Obs. 4.1, 4.3).

4. The provenance of the singularity is opposite (Ver. 5.1, 5.2): 1.0 at $t = 0$ and 250000 at $t = 0.249999$.

5. Where surgery cuts is a convention (Ver. 6.2).

Closing: The two theorems look as though they were performing the same act. But one is mending an injury the world left behind, and the other is mending an injury it inflicted itself. Noticing the likeness is a good thing — not stopping there is a better one.

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