

Symmetry Breaking Is Not One Word

— What Moves: the Lagrangian, the Vacuum, or the Measure - and Nothing Breaks in a Finite System — [Paper
175]

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— *From thinking to syncing.* —

ABSTRACT

The same appearance — the action is symmetric and the answer is not — arrives from three separate places. This paper calls them by name and writes, for each, what must be moved for the breaking to move. **NO NEW PHYSICAL LAW IS CLAIMED** — all of this is standard. First, in spontaneous breaking the action is symmetric and the vacuum is not. At the vacuum $(1, 0)$ of the Mexican hat $V = (|\phi|^2 - 1)^2$ the Hessian eigenvalues are **0.0000000008** and **8.0000000008** — exactly one zero, and it is the Goldstone direction (Section 2). Second, the count is not a choice. For $O(N) \rightarrow O(N - 1)$ the zero eigenvalues number **1, 2, 3, 4** at $N = 2, 3, 4, 5$ — always $N - 1$ (Section 3). Third, explicit breaking moves something else. Putting a non-invariant term $-\varepsilon x$ into the Lagrangian lifts the flat direction, by an amount proportional to ε (the eigenvalue over ε running from **0.999996** to **0.987876**) (Section 4). Fourth, and this is the core and the brake. In a finite system spontaneous breaking does not happen. Solving the double well, the splitting falls from **1.5837** to **0.0385** and never reaches zero, while the ground state stays even however high the barrier (asymmetry 5.4×10^{-14}) (Section 5). Fifth, in an anomaly the action and the vacuum are both symmetric and the measure is not. Under the same transformation with the same integrand, the integral scales by **0.1** against dx and stays at **1.0** against dx/x (Section 6).

Scope of this paper: **NO NEW PHYSICAL LAW IS CLAIMED** — the Nambu–Goldstone theorem, explicit breaking, the Adler–Bell–Jackiw anomaly, Fujikawa's formulation through the measure, and the absence of spontaneous breaking at finite size are all standard. Section 6 is an analogue, not a derivation — the chiral anomaly is not derived, and what is shown is only the structure: the symmetry of the integrand and the symmetry of the measure are independent. What is added is setting the three on one computation, confirming the count $N - 1$ numerically, measuring the pseudo-Goldstone mass against ε , and showing, with eigenvalues and wavefunctions computed from scratch, that a finite system does not break.

1. Notation and position

No field theory is used; only the structure, in the smallest models that carry it. Eigenvalues come from implementations written out here: Jacobi rotations for the small matrices, a Sturm-sequence bisection for the tridiagonal ones.

Position: Paper 85 wrote **that symmetry is the arbiter of what is possible**, and Paper 107 **separated Noether's two theorems**. This paper treats the other side — **the three separate reasons a symmetry stops working**.

2. Spontaneous: the action is symmetric, the vacuum is not

Observation 2.1

$V(\phi) = (|\phi|^2 - 1)^2$ is invariant under $\phi \rightarrow e^{i\alpha}\phi$. Its minima, however, **form a circle**, so **no single one of them is invariant**.

Verification 2.2: The Hessian eigenvalues at the vacuum **(1, 0)**.

eigenvalue	direction
0.000000000800	flat, along the circle (Goldstone)
8.000000000816	radial

(QED)

Verification 2.3: Where the symmetry carries the vacuum.

rotation	image	V there	distance from the original
0.3	(0.9553, 0.2955)	0	0.2989
1.1	(0.4536, 0.8912)	0	1.0454

(QED)

Observation 2.4

Same energy, different point. The symmetry has not gone away — it has carried the vacuum to another vacuum. And no vacuum is fixed by it.

3. The count is not a choice

Verification 3.1: Diagonalise the Hessian at the vacuum of the $O(N)$ -symmetric $V = (\sum x_i^2 - 1)^2$.

N	eigenvalues	zeros	$N - 1$
2	0, 8	1	1
3	0, 0, 8	2	2
4	0, 0, 0, 8	3	3
5	0, 0, 0, 0, 8	4	4

(QED)

Observation 3.2

Always $N - 1$. The number is not chosen: it is the dimension of the group less the dimension of what still fixes the vacuum (the same place at which Paper 86 listed symmetry among the four roots of necessity).

4. Explicit: what moves is the Lagrangian

Verification 4.1: Add the non-invariant $-\epsilon x$. The circle of minima collapses to isolated points.

ε	the lifted eigenvalue	eigenvalue / ε
0	8×10^{-10} (that is, zero)	—
10^{-4}	0.0000999996	0.999996
10^{-3}	0.0009998762	0.999876
10^{-2}	0.0099875403	0.998754
10^{-1}	0.0987875753	0.987876

(QED)

Observation 4.2

At $\varepsilon = 0$ the flat direction is exactly flat. Any ε at all lifts it, and the lift is proportional to ε — a particle that failed to become a Goldstone boson.

Observation 4.3

What moved is the Lagrangian, not the vacuum. Spontaneous and explicit breaking look alike and differ in where a hand was laid.

5. The core: nothing breaks in a finite system

Observation 5.1

Take the same double well as a quantum problem, $H = -d^2/dx^2 + a(x^2 - 1)^2$, and solve it honestly with a Sturm-sequence bisection written out for the purpose.

Verification 5.2

barrier a	E_0	E_1	splitting	ratio to the previous
2	1.739159929	3.322848832	1.5836889027	—
5	3.285153786	4.547352639	1.2621988524	0.7970
10	5.331804619	6.074031419	0.7422267997	0.5880
20	8.227161060	8.477402611	0.2502415507	0.3371
40	12.072664043	12.111168325	0.0385042815	0.1539

Verification 5.3: The parity of the ground state, from the wavefunction by inverse iteration.

barrier a	$\max_x \psi(x) - \psi(-x) $
5	3.6×10^{-15}
20	5.4×10^{-14}

(QED)

Observation 5.4

The splitting falls, and faster as it goes, and never reaches zero. While the barrier is finite the two wells go on talking to one another.

Observation 5.5

And the ground state stays even, however high the barrier. That is: a finite system has no spontaneous symmetry breaking. Breaking is a statement about the infinite-volume limit, and the splitting above is precisely the quantity that has to vanish in it (the structure of Paper 113, where modules are limits rather than walls: what can be said before the limit and after it are different things).

Observation 5.6

So the sentence “the symmetry is broken” carries a silent limit. Said of a finite system, what it means precisely is that the splitting is too small to be observed.

6. Anomalous: what moves is the measure

Observation 6.1

The third case breaks with the action and the vacuum both symmetric. **An anomaly is a Jacobian, and what fails to be symmetric is the measure of the integral** (Fujikawa, 1979).

Verification 6.2: The structure alone, in one dimension. Take $f(x) = e^{-(\log x)^2}$ and compare the integral under $x \rightarrow Lx$ against two measures.

L	$\int f(Lx) \, dx$	$\int f(Lx) \, dx/x$
1	2.27587579447	1.77245385091
2	1.13793789723	1.77245385091
10	0.227587579447	1.77245385091
ratio at $L = 10$	0.1	1.0

(QED)

Observation 6.3

The same transformation, the same integrand, two measures — one integral is untouched and the other is not. The symmetry of what is integrated and the symmetry of what it is integrated against are independent.

Observation 6.4

This is an analogue, not a derivation of the chiral anomaly. What it exhibits is the address — neither the action nor the vacuum has moved; the measure has. Paper 117 met the same place from another angle, listing the anomalous dimension as the counterexample to dimensional analysis.

7. The census

Observation 7.1

kind	what moves	what fails to be symmetric	what is left behind
explicit	the Lagrangian	the symmetry was never there	pseudo-Goldstone, mass $\propto \epsilon$
spontaneous	the choice of vacuum	action symmetric, vacuum not	$N - 1$ Goldstone modes
anomalous	the measure	action and vacuum both symmetric	a classical conservation law fails

Observation 7.2

And spontaneous breaking carries a fourth fence.

finite system	no spontaneous breaking (the ground state is symmetric)
infinite volume	the splitting vanishes, and only then does anything break

Observation 7.3

These three are not a staircase of strength. Which one is happening is settled by where a hand was laid.

8. Conclusion

1. The flat direction of spontaneous breaking is exactly flat (Ver. 2.2): eigenvalues 8×10^{-10} and 8.0.

2. The Goldstone count is $N - 1$ (Ver. 3.1): **four of them at $N = 5$.**
3. The explicit lift is proportional to ε (Ver. 4.1): **0.999996 down to 0.987876.**
4. A finite system does not break (Ver. 5.2, 5.3): **the splitting reaches 0.0385 without reaching zero, and the ground state is even to 5.4×10^{-14} .**
5. An anomaly moves the measure (Ver. 6.2): **0.1 against dx , 1.0 against dx/x .**

Closing: On hearing that a symmetry is broken, the next question is where a hand was laid. Was the equation rewritten, was one answer chosen out of many, or was the counting changed? All three go by the same name and wear the same appearance, and each is repaired differently.

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