

The Delta Is Not a Function but a Map

— Three Families Differing Utterly Pointwise Return the Same Answer to a Test Function — [Paper 178]

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— *From thinking to syncing.* —

ABSTRACT

No function can do what the delta is asked to do. This paper shows that constructively and confirms in numbers that the delta is a map and not a collection of values. **NO NEW MATHEMATICAL THEOREM IS CLAIMED** — all of this is standard since Schwartz. First, a value at one point cannot move an integral. Holding the area at 1 while shrinking the width drives the height from 10 to 10^6 , and the value at $x = 0.01$ drops to 0 along the way — the pointwise limit is not what is wanted (Section 2). Second, and this is the core. At width 0.01 the box, the Gaussian and the sinc take the values **0, 5.399 and 14.472** at $x = 0.02$ — nothing like one another. Tested against a test function, all three converge to $\varphi(0) = 0.367879441171$ (Section 3). Third, what must be moved is therefore the space of test functions. Narrow what one tests against and fewer objects get through — a measure can represent the delta but not its derivative, which returns $-\varphi'(0) = -0.331091497054$ (Section 4). Fourth, the payoff is a derivative that was not there. The difference quotient of the step function runs away, while integrating by parts against a test function gives **0.367879441171** — agreeing with $\varphi(0)$ to 1.2×10^{-13} (Section 5). Fifth, the fence is that the product is not defined. Testing the square of the approximating family gives **1.033, 10.377, 103.777**, and diverges (Section 6).

Scope of this paper: **NO NEW MATHEMATICAL THEOREM IS CLAIMED** — Schwartz's theory of distributions, delta sequences, the distributional derivative, and the fact that distributions cannot in general be multiplied (Schwartz's impossibility theorem, 1954) are all standard. Nor is Dirac criticised — no framework for distributions existed in 1927, and the object he used was made rigorous afterwards. What is added is constructing in numbers the claim that no function can do it, setting three approximating families side by side to show that they differ utterly pointwise and agree against a test function, actually computing the distributional derivative of the step by parts, and measuring how fast the square diverges.

1. Notation and position

The **test function** used throughout is smooth with compact support,

$$\varphi(x) = e^{-1/(1-x^2)}(1 + 0.9\,x) \quad (|x| < 1), \quad \varphi(x) = 0 \quad (|x| \geq 1).$$

It is deliberately not symmetric, so that the mistake of simply evaluating at the origin cannot agree by accident. Here $\varphi(0) = 1/e = 0.367879441171$.

Position: Paper 155 treated spaces with no points and showed that points cannot be recovered from the open sets. This paper is its pair — here there is nothing at the point, and an object all the same.

2. A value at one point cannot move an integral

Observation 2.1

Two things are demanded of the delta at once — **that it vanish away from the origin and that its integral be 1.**

Observation 2.2

A function carrying a value at a single point contributes **nothing to an integral**, however large the value. **Getting integral 1 takes width, and with width it no longer vanishes off the origin. A function cannot meet both demands.**

Verification 2.3: Shrink the width while holding the area at 1.

width w	height $1/w$	integral	value at $x = 0.01$
0.1	10.0	1.0	10.0
0.01	100.0	1.0	0.0
0.001	1000.0	1.0	0.0
10^{-6}	10^6	1.0	0.0

(QED)

Observation 2.4

The integral stays at 1 while the values run to infinity at one point and to 0 everywhere else. What this family converges to pointwise is not the thing being sought.

3. The core: utterly different pointwise, identical against a test

Verification 3.1: Three families of area 1, all at width 0.01.

x	box	Gaussian	sinc
0	100.0	39.894228	31.830989
0.02	0.0	5.3990967	14.471918
0.13	0.0	$7.9988278 \times 10^{-36}$	1.0287948

Observation 3.2

At $x = 0.02$ the box gives 0, the Gaussian 5.4 and the sinc 14.5. As functions they are nothing like one another. And the sinc still stands at 1.03 at $x = 0.13$: it does not even converge pointwise.

Verification 3.3: The same three, tested against φ .

width	box	Gaussian	sinc
0.1	0.367572644909	0.36414456767	0.362742785466
0.01	0.367876375486	0.367842647708	0.367880239715
0.001	0.367879410515	0.367879073291	0.359945061448

The value to compare against is $\varphi(0) = \mathbf{0.367879441171}$. (QED)

Observation 3.4

All three head for the same number, and that number is $\varphi(0)$. The sinc converges slowly and oscillates (hovering near 0.36) — which is to say that a family failing to converge pointwise can still have the same limit against a test function.

Observation 3.5

So the object is not a collection of values. It is what the values do to a test function, and on that the three agree. The delta is the map $\varphi \mapsto \varphi(0)$ itself.

4. What must be moved is the space of test functions

Observation 4.1

Change what φ is allowed to be and the family of objects that can act on it changes with it.

test functions	what may act	note
smooth, compact support	distributions	the widest class
smooth, rapidly decreasing	tempered distributions	Fourier transform stays inside
continuous, compact support	measures	the delta's derivative is excluded
polynomials only	very little	too few tests to separate anything

Verification 4.2: The derivative of the delta separates what a coarser test cannot.

acting on φ	the delta	its derivative
the number returned	0.367879441171	−0.331091497054

The derivative of the delta returns $-\varphi'(0)$. (QED)

Observation 4.3

A measure can represent the delta and cannot represent the object returning $-\varphi'(0)$. One symbol names a different object depending on what it is tested against — that is the separator.

5. The payoff: a derivative that was not there

Verification 5.1: The difference quotient of the step function H .

h	$(H(h) - H(-h))/(2h)$
0.1	5.0
0.01	50.0
10^{-6}	500000.0

It runs away: classically there is no derivative at the origin. (QED)

Verification 5.2: Yet computing $-\int H\varphi'$ by parts gives —

$-\int H\varphi' dx$	0.367879441171
$\varphi(0)$	0.367879441171
difference	1.22626×10^{-13}

(QED)

Observation 5.3

The distributional derivative of the step acts exactly as the delta does. The derivative did not exist and now it does — because the question moved from values to action.

Observation 5.4

This payoff sits next to the limit Paper 177 described — there the Cantor function failed the hypothesis of differentiability everywhere and the fundamental theorem broke. Here a function that is not differentiable is given a derivative. Both turn on which sense of “derivative” is meant.

6. The fence: the product is not defined

Observation 6.1

Distributions form a vector space and can be differentiated any number of times. In general they cannot be multiplied.

Verification 6.2: Test the square of the approximating family.

width	$\int (\delta_n)^2 \varphi$
0.1	1.032540663
0.01	10.37716851
0.001	103.7768225

It grows like the reciprocal of the width and does not converge. (QED)

Observation 6.3

“Delta squared” is not a distribution. The fence is that the product is undefined, which is why this definition buys differentiation and not an algebra (Schwartz's impossibility theorem of 1954 says that the obstruction is not an accident).

Observation 6.4

This is the same shape of bargain as in Paper 168 — there, closure under limits was bought and absolute convergence paid. Here, differentiation without end is bought and multiplication paid.

7. The census

Observation 7.1

object	what it is	what it can do	what it cannot
a function	values at points	multiply and differentiate	may fail to be differentiable
a measure	a number on each set	the delta is a measure	its derivative is not a measure
a distribution	a number on each test function	differentiate without end	no product

Observation 7.2

These three are not a staircase of strength — a function can do things a distribution cannot, and the reverse. What settles it is what one tests against.

8. Conclusion

1. A value at one point cannot move an integral (Ver. 2.3): height 10^6 , and 0 at $x = 0.01$.
2. Utterly different pointwise (Ver. 3.1): 0, 5.399, 14.472 at $x = 0.02$.
3. Identical against a test function (Ver. 3.3): all three to $\varphi(0) = 0.367879441171$.
4. The delta's derivative returns $-\varphi'(0)$ (Ver. 4.2): -0.331091497054 , beyond any measure.
5. The step is given a derivative (Ver. 5.2): difference 1.2×10^{-13} .
6. The square diverges (Ver. 6.2): 1.03 to 103.78.

Closing: Describing the delta as an infinitely tall, infinitely thin function is a kind picture and looks in the wrong place. What to look at is neither the height nor the thinness but what it returns to a test function. Look there, and

three families with nothing in common are seen, in numbers, to have been naming one object all along.

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