

The Yamagishi Kinetic Energy Equation

– $E_k = 2\pi v^2$, and the Three Distinct Roots of 2π – [Paper 18]

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— *From thinking to syncing.* —

Abstract

Take the classical kinetic energy $E_k = \frac{1}{2}mv^2$ and substitute the Yamagishi patch obtained from the $n = 2$ uniqueness theorem, $m = 4\pi = \text{Vol}(S^2)$: the result is $E_k = 2\pi v^2$. **The substitution is correct.** The coefficient $\frac{1}{2}$ is the result of the integration $\int_0^v v' dv' = \frac{v^2}{2}$. **This paper examines the identity of the 2π appearing in this equation.** The number 2π occurs at several places in this series, but **they are the same number, not the same root.** This paper calls the roll of **three distinct roots: Root A**, the solid angle of a hemisphere ($\Omega_{\text{hemi}} = 2\pi$); **Root B**, the periodicity of phase ($\sin(x + 2\pi) = \sin(x)$; in $\hbar = 2\pi\hbar$ and in the phase-space cell $d^3x d^3p / (2\pi\hbar)^3$, the 2π also descends from the periodicity of the phase $e^{i\theta}$, and so **belongs to this Root B**); and **Root C**, $\text{Vol}(S^1)$ — a value on the odd staircase (Paper 115). **None of the three carries mass dimension. And the 2π of this paper is none of them.** In E_k the factor in front sits in the mass slot, and $m = 4\pi$ is a mass value placed under a choice of units (Papers 35 and 60), so the 2π of this paper is **half of that value**, and lines up with the solid angle of a hemisphere **as the same number.** **Root C gives a structural diagnosis of the reading of 2π as the circumference of S^1 :** $\frac{1}{2}\text{Vol}(S^2) = \text{Vol}(S^1)$ is numerically correct, but $\text{Vol}(S^2)$ lives on the even staircase and $\text{Vol}(S^1)$ on the odd one, and $4\pi \rightarrow 2\pi$ is not a step of the recursion (after 4π comes $\frac{8\pi^2}{3}$; after 2π comes $2\pi^2$). The agreement is therefore **a signature, not a connection by a generating rule** (Paper 100). Further, the paper shows that **the relativistic expansion $E = 4\pi c^2 + 2\pi v^2 + \frac{3}{2}\pi \frac{v^4}{c^2} + \dots$ is a power series in the dimensionless v/c , where the “module boundary” of Paper 113 resides; that the v^2 here is a quantity in the flat approximation of velocity space** (the hyperbolic geometry of Papers 60 and 63 appears as $v \sim c$); and that **this equation is the $d = 3$ member of a family with one member per dimension** (the ruler of Paper 114; for $d = 4$ one gets $E_k = \pi^2 v^2$).

Scope of this paper (scope note): This paper offers a **structural interpretation** of established physical law. The algebraic relations presented are substitutions into known expressions; no new physical law and no prediction is claimed. The substitution $m = 4\pi$ and the like constitute a **choice of natural units**, possible in the same sense as $G = 1$ (geometrised units) or SI units. Physics does not determine which is “correct”; what this paper exhibits is **how the equations look once this unit system is adopted** (conversion factors are required to carry them into SI). The coefficient $\frac{1}{2}$ is treated as the result of an integration, and no further explanation of it is claimed. The appearance of the same number 2π in several contexts is not treated as evidence of a common origin (Papers 48, 86, 100, 115). The agreement with Papers 17, 19 and 20 descends from the same substitution and is a consistency check, not an independent confirmation. Layer-3 lenses are quarantined with a non-scientific label. Relation to earlier papers: the uniqueness of 4π comes from Paper 01; that in $m = 4\pi$ the 4π is a conventional value placed in the mass slot, from Papers 35 and 60; the two staircases, from Paper 115; module boundaries, from Paper 113; the hyperbolic geometry of velocity space, from Papers 60 and 63; and the ruler of whether a dimension is selected, from Paper 114. **What this paper adds** is the roll call of the three distinct roots of 2π (the solid angle of a hemisphere, the period of a phase, the value on the odd staircase), and the distinction that **in $E_k = 2\pi v^2$ the 2π is none of them, and lines up with the three as the same number, being half of a value placed in the mass slot.**

1. Introduction: The Coefficient $\frac{1}{2}$ Is the Result of an Integration

In classical mechanics the kinetic energy is derived as

$$E_k = \int_0^v m v' dv' = \frac{1}{2} m v^2$$

The coefficient $\frac{1}{2}$ is the result of this integration: $\int_0^v v' dv' = \frac{v^2}{2}$ is the elementary fact of calculus that integrating x^1 yields $\frac{x^2}{2}$.

What this paper does, therefore, is not to **explain** $\frac{1}{2}$ but to **record** that, once $m = 4\pi$ is adopted, it can be written $\frac{1}{2} \times 4\pi = 2\pi$ — and to **identify the 2π that thereby appears.**

By the $n = 2$ uniqueness theorem (Paper 01), in $\mathbb{R}^3$ the condition for the shell-mass growth rate of a density $\rho(r) = r^{-n}$ to be independent of r is exactly $n = 2$, and the constant value is then $\text{Vol}(S^2) = 4\pi$. This paper substitutes that 4π for the mass.

2. The Yamagishi Kinetic Energy Equation

Theorem 2.1 (Yamagishi Kinetic Energy Equation)

Under the Yamagishi patch $m = 4\pi$, the kinetic energy of a point particle can be written

$$E_k = \frac{1}{2}mv^2 = \frac{1}{2}(4\pi)v^2 = 2\pi v^2$$

This is a substitution, not a proof and not a discovery. **What is fixed is this identity alone.** The sections that follow write out what the 2π appearing here is, and is not.

3. What Is 2π ? — A Roll Call of Three Distinct Roots

The number 2π occurs at several places in this series. **But they are the same number, not the same root.** This section separates them into three, and then looks at where the 2π of this paper sits.

3.1 Root A: The solid angle of a hemisphere

Established Fact 3.1 (The solid angle of a hemisphere)

$$\Omega_{\text{full}} = \int_0^{2\pi} d\varphi \int_0^\pi \sin \theta d\theta = 2\pi \cdot 2 = 4\pi, \quad \Omega_{\text{hemi}} = \int_0^{2\pi} d\varphi \int_0^{\pi/2} \sin \theta d\theta = 2\pi \cdot 1 = 2\pi$$

so 2π is precisely the solid angle of a hemisphere.

4π (the full sphere) and 2π (the hemisphere) are both solid angles and carry no dimension. The 2π appearing in solid-angle integrals over a hemisphere is this Root A.

3.2 Root B: The periodicity of phase ($\hbar = 2\pi\hbar$ belongs here too)

2π also appears as the period of a phase: $\sin(x + 2\pi) = \sin(x)$. In the period of a simple pendulum, $T = 2\pi\sqrt{L/g}$, the 2π is of this kind.

Observation 3.2 ($\hbar = 2\pi\hbar$ is not a new root)

What appears in the relation $\hbar = 2\pi\hbar$ as 2π , and what appears in the counting of phase-space cells

$$\frac{d^3x\,d^3p}{(2\pi\hbar)^3}$$

as 2π (the $\hbar^{-3}$ of the Stefan–Boltzmann constant in Paper 43; address B of Paper 112), **descend from the periodicity of the phase $e^{i\theta}$** . That is, **this is Root B, not a new fourth root**.

Not multiplying roots carelessly is itself part of the roll call. What is to be counted is not “the places where 2π appears” but “the independent mechanisms that produce 2π ”.

3.3 Root C: $\text{Vol}(S^1)$ — a value on the odd staircase (Paper 115)

Paper 115 recorded that, because the sphere-volume recursion

$$\text{Vol}(S^n) = \frac{2\pi}{n-1} \text{Vol}(S^{n-2})$$

steps by two, parity is preserved and the staircase splits into **two chains, even and odd**.

Even staircase	$\text{Vol}(S^0) = 2 \rightarrow \text{Vol}(S^2) = 4\pi \rightarrow \text{Vol}(S^4) = \frac{8\pi^2}{3} \rightarrow \dots$
Odd staircase	$\text{Vol}(S^1) = 2\pi \rightarrow \text{Vol}(S^3) = 2\pi^2 \rightarrow \text{Vol}(S^5) = \pi^3 \rightarrow \dots$

$\text{Vol}(S^2) = 4\pi$ lives on the even staircase; $\text{Vol}(S^1) = 2\pi$ lives on the odd one. This is Root C.

3.4 Where does the 2π of this paper sit? — the mass slot (Papers 35 and 60)

Observation 3.3 (In $E_k = 2\pi v^2$, the 2π sits in the mass slot)

Since $[E_k] = ML^2T^{-2}$ while $[v^2] = L^2T^{-2}$, the factor standing in front in $E_k = 2\pi v^2$ carries mass. Paper 60 did the same bookkeeping for $E = 4\pi c^2$ and showed that its 4π is not a solid angle but a mass value placed under a choice of units — a convention (Paper 35 likewise places $m = 4\pi$ as a choice of natural units).

The 2π of this paper is therefore the conventional value $m = 4\pi$ multiplied by the $\frac{1}{2}$ of the integration, and is none of Roots A, B and C. None of the three roots carries mass dimension, and none can stand in the mass slot.

The 2π of this paper and the 2π of the hemisphere's solid angle line up as the same number. One can view $4\pi \rightarrow 2\pi$ as “from the full sphere to the hemisphere”, but that is an overlay of form, not an identification by dimension.

The 2π of this paper is not counted as a fourth root. A root is an independent mechanism that produces 2π , and the 2π here is not a value produced by a mechanism but half of a value placed by choosing units.

3.5 The reading as the circumference of S^1 — a diagnosis via Paper 115

The reading of 2π as “the circumference of S^1 ” lacks a connection for two reasons. The first is **dimension**: the circumference of S^1 is a length, whereas the 2π of this paper sits in the mass slot. Second, **the staircases of Paper 115** supply a more structural reason.

Corollary 3.4 (A numerical agreement is not a connection by a generating rule)

$\frac{1}{2} \text{Vol}(S^2) = \text{Vol}(S^1)$ is numerically correct ($\frac{1}{2} \cdot 4\pi = 2\pi$). But $\text{Vol}(S^2)$ and $\text{Vol}(S^1)$ are residents of separate staircases, and $4\pi \rightarrow 2\pi$ is not a step of the recursion: after 4π comes $\frac{8\pi^2}{3}$, and after 2π comes $2\pi^2$.

The agreement is therefore a **signature** — two values on different chains happening to coincide numerically — and not a connection by a generating rule. A signature must not be mistaken for a root (Paper 100).

3.6 The roll call of the three roots and the 2π of this paper

Root	What the 2π is	Origin	Where it appears
A	Solid angle of a hemisphere	Half of the full sphere 4π (geometry)	Solid-angle integrals over a hemisphere
B	Fundamental period of a phase	Periodicity of $e^{i\theta}$ (analysis)	Pendulum $T = 2\pi\sqrt{L/g}$; $\hbar = 2\pi\hbar$; phase-space cell (Papers 43, 112)
C	$\text{Vol}(S^1)$	A value on the odd staircase (Paper 115)	The step before $\text{Vol}(S^3) = 2\pi^2$
This paper	Half of a mass value	The convention $m = 4\pi$ (Papers 35, 60) and the integration's $\frac{1}{2}$	$E_k = 2\pi v^2$ (the same number as the three)

The three roots and the 2π of this paper are not merged (Papers 48, 86, 100). That the same 2π appears in different contexts does not mean they are the same thing.

4. The Relativistic Expansion and the Module Boundary (Paper 113)

Expanding Einstein’s total energy $E = \gamma mc^2$ at low speed and substituting $m = 4\pi$:

Theorem 4.1 (Relativistic expansion)

$$E = (4\pi)c^2 + 2\pi v^2 + \frac{3}{2}\pi \frac{v^4}{c^2} + \cdots$$

The first term is the rest energy $4\pi c^2$; the second is the $E_k = 2\pi v^2$ of this paper.

Observation 4.2 (This is a power series in v/c)

The expansion above comes from the series for $\gamma = (1 - v^2/c^2)^{-1/2}$, so **the expansion parameter is the dimensionless v/c** . Paper 113 recorded that the **boundary between an older and a newer theory always lies where a dimensionless number crosses 1**, and that Newtonian mechanics is a **limit** of relativity (as $c \rightarrow \infty$ the Poincaré group contracts to the Galilei group).

The $E_k = 2\pi v^2$ of this paper is the second term of that series — a term of the regime $v/c \ll 1$. It is not a different law behind a wall, but **a low-order term of the same series**.

5. Where Does This v^2 Live? (Papers 60 and 63)

The v^2 here is a quantity in **velocity space**. Papers 60 and 63 treated the fact that, in special relativity, velocity space is a **hyperbolic geometry of curvature $K = -1/c^2$** .

Observation 5.1 (v^2 as a flat approximation)

For $v \ll c$ velocity space is **very nearly flat**, and the quadratic form there is $\frac{1}{2}mv^2$. **As $v \sim c$ the hyperbolic structure of velocity space emerges and velocities no longer add simply** (rapidity becomes the hyperbolic distance).

That is, **the $E_k = 2\pi v^2$ of this paper is an expression within the flat approximation of velocity space**.

6. Dimensional Roll Call: Does This Equation Select a Dimension? (Paper 114)

Paper 114 set up the ruler “**does the object know what dimension it is in?**” and recorded that this series’ **$n = d - 1$ uniqueness is a family with one member per dimension**, belonging to the **dimension-agnostic** side.

Corollary 6.1 (The equation here is likewise a family)

In $\mathbb{R}^d$, adopting $m = \text{Vol}(S^{d-1})$ gives

$$E_k^{(d)} = \frac{1}{2} \text{Vol}(S^{d-1}) v^2$$

so that $d = 3$ yields $E_k = 2\pi v^2$ and $d = 4$ yields $E_k = \pi^2 v^2$ (from $\frac{1}{2} \cdot 2\pi^2$). The equation of this paper is the $d = 3$ member of that family.

It is not that 2π appears because three dimensions are special. 2π appears because at $d = 3$ the value of $\text{Vol}(S^{d-1})$ is 4π .

7. Consistency with Papers 17, 19 and 20

7.1 Paper 17 (the kinetic energy of a plane wave)

In Paper 17 the kinetic-energy expectation value of a plane wave is

$$\langle E_k \rangle = \frac{k^2}{8\pi} = \frac{(4\pi v)^2}{8\pi} = 2\pi v^2$$

in agreement with the present paper. The agreement holds because **both descend from the same substitution $m = 4\pi$** ; it is a consistency check, not an independent confirmation.

7.2 Paper 19 (Yamagishi spherical compression pressure)

Into the gas pressure $P = \frac{1}{3}nmv^2$, substituting $m = 4\pi$ gives

$$P = \frac{1}{3}n(4\pi)v^2 = \frac{4}{3}\pi nv^2 = \frac{2}{3}n(2\pi v^2)$$

so that it can be written as **the kinetic-energy density per unit volume $n(2\pi v^2)$, multiplied by $\frac{2}{3}$** .

7.3 Paper 20 (temperature) — the power of v is fixed

To the $E_k = 2\pi v^2$ of this paper, applying the standard equipartition theorem $\langle E_k \rangle = \frac{3}{2}k_B T$ gives

$$kT = \frac{2}{3}E_k = \frac{4}{3}\pi v^2$$

which **agrees exactly with the result of Paper 20** (verified numerically at $v = 0.5, 1, 2, 3.7$).

This consistency also fixes the power of v . From $E_k = 2\pi v^2$ no v^3 can come — the $kT = \frac{4}{3}\pi v^3$ of the first edition of Paper 20 was found by this consistency check and corrected to v^2 .

The author's personal reflection (Layer 3; labelled non-scientific / not a claim about mechanism)

Through the lens of this framework one is tempted to read the passage $4\pi \rightarrow 2\pi$ as “**compression into a hemisphere (a mirror fold)**” — the operation by which the Universe OS uses only half of the full solid angle when packing energy. That 2π is the same number as the solid angle of a hemisphere is a fact.

But **it is an interpretation, not a derivation**. The coefficient $\frac{1}{2}$ arises from an integration, and in E_k the 2π in front sits in the mass slot. “Compression into a hemisphere” is **a reading laid over the form**, and the order is “**the integration produced $\frac{1}{2}$, and a geometric reading was added to that result**”, not the reverse.

Seeing 2π as the circumference of S^1 is not entirely groundless either — $\text{Vol}(S^1)$ genuinely is 2π . One goes wrong **when that agreement is read as a connection**. The staircases of Paper 115 gave that distinction a precise name. **That two numbers are equal and that they come from the same place are different things**. One stops short of the cliff (Paper 101).

8. Conclusion

Adopting $m = 4\pi$, the kinetic energy can be written $E_k = 2\pi v^2$, consistently with Papers 17, 19 and 20 (Sections 2 and 7). **The 2π in this equation is the same number as each of the three distinct roots — the solid angle of a hemisphere (Root A), the period of a phase (Root B), the odd-staircase value $\text{Vol}(S^1)$ (Root C) — but is none of them.** It is half of the conventional value 4π placed in the mass slot (Section 3). The three roots **come from separate places: geometry, analysis and the staircase**, and the agreement $\frac{1}{2} \text{Vol}(S^2) = \text{Vol}(S^1)$ is a signature, not a connection. The equation is a low-order term of a power series in v/c (Section 4), a quantity of the flat approximation of velocity space (Section 5), and the $d = 3$ member of a family with one member per dimension (Section 6).

That two numbers are equal and that they come from the same place are different things. The 2π of this paper is an example that checks this distinction on a single equation.

Revision Record

4th edition (September 2026): brought the format into line with the current template (the positioning and the note on units are gathered in the scope note directly after the abstract, and the history of the editions is moved to this record). Removed the edition label from the subtitle. **In $E_k = 2\pi v^2$, the identity of the 2π is revised in line with Papers 60 and 35:** the 3rd edition identified it with Root A (the solid angle of a hemisphere), but the factor in front in E_k sits in the mass slot and $m = 4\pi$ is a value placed under a choice of units, so the 2π of this paper is half of that value and lines up with the solid angle of a hemisphere as the same number (Section 3.4). Removed a sentence naming earlier papers as treating the period of a phase (the papers named no longer take it as their subject). Self-citations now carry their actual titles and DOIs, and external references are given as author, year, title and journal.

3rd edition (August 2026): extended the roll call of distinct roots of 2π from two to three, adding Root C ($\text{Vol}(S^1)$) by way of the two staircases of Paper 115. Wrote that in $\hbar = 2\pi\hbar$ the 2π belongs to Root B, and described the boundary of the relativistic expansion (Paper 113), the flat approximation of velocity space (Papers 60 and 63), and the dimensional status of the equation (Paper 114).

2nd edition (July 2026): withdrew the first edition's reading "a projection onto the circumference of S^1 " for 2π , on the ground that a length and a solid angle differ in dimension, and made explicit that the coefficient $\frac{1}{2}$ is the result of an integration.

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