

“The Curse of Dimensionality” Is Not One Curse

— One Concentration Giving Both the Curse and the Blessing: the Separator Is Whether You Wanted the
Function to Vary — [Paper 182]

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— *From thinking to syncing.* —

ABSTRACT

This corpus has been writing $\text{Vol}(S^{d-1})$ since Paper 01. It has never once remarked that the ball this solid angle measures vanishes as the dimension grows. This paper opens that — and finds that the phrase “curse of dimensionality” covers several things with different causes, of which the sharpest is not a curse at all. **NO NEW MATHEMATICAL THEOREM IS CLAIMED.** First, the volume of the unit ball peaks at $d = 5$ and falls thereafter, reaching 2.37×10^{-40} at $d = 100$ (Section 2). Second, where the volume went is two separate statements. Within one body the mass gathers in a thin skin (0.99996 of it within 0.01 of the surface at $d = 1000$); between two bodies the ball loses to the cube outright (1.87×10^{-70} at $d = 100$) (Section 3). Third, two random directions are nearly orthogonal. The mean absolute cosine is 0.0252 at $d = 1000$, matching $\sqrt{2/(\pi d)} = 0.0252$ (Section 4). Fourth, this is what people mean by the curse. The relative gap between the nearest and the farthest point collapses to 0.113 at $d = 1000$, so “nearest” stops selecting anything (Section 5). Fifth, and this is the core. Read the other way, that same concentration is the blessing. Projecting 300 points from 1000 dimensions down to 100 at random moves all 44850 pairwise distances by 5.5% on average — the curse and the blessing are the same theorem, and what separates them is only whether you wanted the function to vary (Section 6). Sixth, the fence is what the concentration is around. The maximum of d standard gaussians does concentrate, but around a value that keeps growing (Section 7).

Scope of this paper: NO NEW MATHEMATICAL THEOREM IS CLAIMED — the volume of the ball, concentration of measure, the Johnson–Lindenstrauss lemma, and the degeneration of nearest neighbours in high dimension are all standard. No machine-learning method is evaluated. On Papers 94 and 114: 94 split the word dimension into topological, fractal and special integer dimensions; 114 sorted objects by whether they know their own dimension. This paper asks what happens when the dimension is large — a different question from either. On Paper 139: that paper also mentions the curse once, but about the convergence rate of empirical measures, which is a different quantity from the geometry here. What is added is separating five things by what each is a statement about, setting the curse and the blessing side by side in numbers as two readings of one concentration, measuring the fence that concentration is around the mean and not around a constant, and writing down where the $\text{Vol}(S^{d-1})$ this corpus keeps carrying actually goes.

1. Notation and position

Write $V_d = \pi^{d/2}/\Gamma(d/2 + 1)$ for the volume of the unit ball in d dimensions and $A_d = 2\pi^{d/2}/\Gamma(d/2)$ for the area of the unit sphere S^{d-1} . Everything sampled was sampled for this paper, with the sample size stated.

Position: From the $n = d - 1$ uniqueness of Paper 01 through the exponent formula of Paper 116 to the solid angles of Paper 36, this corpus has written $\text{Vol}(S^{d-1})$ over and over. What that value does as d grows has never been asked.

2. Where the volume this corpus carries actually goes

Verification 2.1

d	$A_d = \text{Vol}(S^{d-1})$	V_d (volume of the unit ball)
3	12.56637061	4.188790205
5	26.31894507	5.263789014
6	31.00627668	5.16771278
7	33.07336179	4.72476597
10	25.5016404	2.55016404
20	0.5161378278	0.02580689139
50	$8.651096229 \times 10^{-12}$	$1.730219246 \times 10^{-13}$
100	$2.368202102 \times 10^{-38}$	$2.368202102 \times 10^{-40}$

(QED)

Observation 2.2

The ball's volume peaks at $d = 5$ and falls from there, while the solid angle peaks at $d = 7$ — even the two peaks sit in different places. And by $d = 100$ the $\text{Vol}(S^{d-1})$ this corpus keeps writing is down in the 10^{-38} range.

3. Where the volume went is two separate statements

Verification 3.1: The fraction of a ball's volume lying within ϵ of its surface, which is exactly $1 - (1 - \epsilon)^d$.

d	$\varepsilon = 0.1$	$\varepsilon = 0.01$	$\varepsilon = 0.001$
2	0.19	0.0199	0.001999
10	0.65132156	0.095617925	0.0099551198
100	0.99997344	0.63396766	0.095207853
1000	1.0	0.99995683	0.63230458
10000	1.0	1.0	0.99995483

(QED)

Verification 3.2: The ball as a fraction of the cube of side 2.

d	$V_d/2^d$
2	0.78539816
5	0.16449341
10	0.0024903946
20	2.461137×10^{-8}
100	$1.8681821 \times 10^{-70}$

(QED)

Observation 3.3

These are different statements. The first says where the mass of one body sits and follows from one line of algebra once ε is fixed. The second compares two bodies and follows from the ball having no corners. Calling both the curse of dimensionality hides the difference.

4. Two random directions are nearly orthogonal

Verification 4.1: In each dimension take 20000 pairs of random unit vectors and measure the absolute cosine of the angle between them.

d	mean $ \cos \theta $	$\sqrt{2/(\pi d)}$	largest over 20000 pairs
3	0.49988973	0.46065887	0.99998180
10	0.25722537	0.25231325	0.93703595
100	0.07953272	0.07978846	0.38790477
1000	0.02521252	0.02523133	0.14522426

(QED)

Observation 4.2

The typical cosine falls like $1/\sqrt{d}$. In a thousand dimensions two vectors drawn at random are orthogonal for all practical purposes. This will return later on the side of the blessing.

5. What people mean by the curse

Verification 5.1: Take 2000 points uniformly in the unit cube and look at their distances from a random query point.

d	nearest	farthest	relative gap
1	0.000287	0.759371	2644.59
5	0.239713	1.440486	5.009207
20	0.940906	2.617093	1.781461
100	3.304831	4.780400	0.446489
1000	12.168306	13.539749	0.112706

(QED)

Observation 5.2

The nearest and the farthest point are at almost the same distance. The operation “take the nearest” stops selecting anything — this is what the curse actually is.

6. The core: read the other way, the same concentration is the blessing

Observation 6.1

What flattened the distances in Section 5 is the fact that a **Lipschitz function of many independent coordinates is nearly constant. But that same fact also means distances survive a projection.**

Verification 6.2: Take **300** points in **1000** dimensions (**44850** pairs), project to k dimensions with a random matrix, and measure the ratio of every pairwise distance.

k	worst distortion	mean distortion	compression
20	0.624735	0.119932	50×
50	0.457538	0.078553	20×
100	0.300259	0.055155	10×
200	0.194477	0.038989	5×
400	0.160552	0.029956	2.5×

(QED)

Observation 6.3

A thousand dimensions crushed into a hundred, and 44850 distances move by 5.5% on average. What makes this possible is the same concentration that destroyed the nearest neighbour in Section 5. The curse and the blessing are one theorem, and what separates them is only whether the function was wanted to vary.

Observation 6.4

If the aim is to pick a neighbour by distance, not varying is a disaster. If the aim is to compress while keeping distances, not varying is the whole point. One phenomenon, two verdicts — as when Paper 95 separated convention from fact, the verdict is not a property of the phenomenon.

7. The fence: what it concentrates around is not a constant

Observation 7.1

A common misreading is that in high dimension everything settles onto a single value independent of d . The maximum of d standard gaussians is a 1-Lipschitz function and so does concentrate — around a value that keeps rising.

Verification 7.2: 4000 draws in each dimension.

d	mean of the max	$\sqrt{2 \ln d}$	ratio	spread over mean
10	1.549540	2.145966	0.7221	0.3830
100	2.501465	3.034854	0.8242	0.1721
1000	3.238935	3.716922	0.8714	0.1067
10000	3.855180	4.291932	0.8982	0.0779
100000	4.382860	4.798526	0.9134	0.0614

The two columns are not equal — $\sqrt{2 \ln d}$ is the leading growth rate rather than the value, and the ratio approaches 1 slowly. (QED)

Observation 7.3

The spread relative to the mean shrinks from 0.383 to 0.0614, so it certainly concentrates. But the value it concentrates on climbs from 1.55 to 4.38. “Concentrates” is a statement about the spread, not about the location.

Verification 7.4: And without a Lipschitz bound nothing at all is promised. Take the product of the coordinates, whose Lipschitz constant blows up.

d	spread over mean	concentrates?
2	1.202000	no
5	3.119326	no
10	10.070772	no
20	18.666851	no

The relative spread grows with d . (QED)

Observation 7.5

Concentration is a theorem about Lipschitz functions and not about high dimension as such. The fence is the Lipschitz constant.

8. The census

Observation 8.1

how it is called	what it is a statement about	quantity	curse or blessing
mass in the skin	inside one body	$1 - (1 - \epsilon)^d$	neither
ball loses to cube	two bodies compared	$V_d/2^d$	grids stop working
nearly orthogonal	two random points	$1/\sqrt{d}$	blessing
nearest loses meaning	many random points	relative gap	curse
samples grow exponentially	a covering count	ϵ^{-d}	curse

Observation 8.2

These five are not a staircase. The first two are geometric identities and need no randomness. The next two are statements about random points and need probability. The last is a count of an algorithm's work and is not geometry at all. Because one phrase covers them, one remedy seems to be called for.

9. Conclusion

1. The unit ball's volume peaks at $d = 5$ (Ver. 2.1): 2.37×10^{-40} at $d = 100$.
2. The solid angle peaks at $d = 7$, so even the peaks differ (Obs. 2.2).
3. The skin and the cube are different statements (Obs. 3.3).
4. Two random directions are nearly orthogonal (Ver. 4.1): 0.0252 at $d = 1000$, matching prediction.
5. The nearest neighbour loses meaning (Ver. 5.1): a relative gap of 0.113.
6. The same concentration becomes the blessing (Ver. 6.2): 5.5% mean distortion at tenfold compression.
7. What it concentrates on is not a constant (Ver. 7.2): spread down to 0.0614 while the value climbs to 4.38.

Closing: When someone says “the curse of dimensionality”, the next question is what it is a statement about. Inside one body, between two bodies, about random points, or about an algorithm's work. And the sharpest of them is not a curse — the same concentration that makes distances unusable for selection is what allows dimension to be thrown away while distances are kept. One phenomenon, and what makes it a curse or a blessing is what we happened to want.

References

1. Lévy, P. (1951). *Problèmes concrets d'analyse fonctionnelle*. Paris: Gauthier-Villars.
2. Bellman, R. (1961). *Adaptive Control Processes: A Guided Tour*. Princeton: Princeton University Press.
3. Johnson, W. B., Lindenstrauss, J. (1984). *Extensions of Lipschitz Mappings into a Hilbert Space*. Contemporary Mathematics, 26.
4. Milman, V. D., Schechtman, G. (1986). *Asymptotic Theory of Finite Dimensional Normed Spaces*. Berlin: Springer.
5. Beyer, K., Goldstein, J., Ramakrishnan, R., Shaft, U. (1999). *When Is Nearest Neighbor Meaningful?*. Proceedings of the 7th International Conference on Database Theory.
6. Ledoux, M. (2001). *The Concentration of Measure Phenomenon*. Providence: American Mathematical Society.
7. Yamagishi, Y. (2026). “Dimension” Is Not One Thing [Paper 94]. Zenodo. 10.5281/zenodo.21758581
8. Yamagishi, Y. (2026). *Which Objects Know They Are in Four Dimensions?* [Paper 114]. Zenodo. 10.5281/zenodo.21963544

9. Yamagishi, Y. (2026). *What Does the Exponent of $\mathbf{c}$ Count?* [Paper 116]. Zenodo. 10.5281/zenodo.22003794
10. Yamagishi, Y. (2026). *Three Distinct Roots of Aggregation* [Paper 124]. Zenodo. 10.5281/zenodo.22057728
11. Yamagishi, Y. (2026). *Measuring Overlap, or Measuring Displacement* [Paper 139]. Zenodo. 10.5281/zenodo.22081801
12. Yamagishi, Y. (2026). *The Order of the Quantifiers Is the Whole Content* [Paper 177]. Zenodo. 10.5281/zenodo.22098103
13. Yamagishi, Y. (2026). *The Multiplication Does Not Decide the Second System* [Paper 181]. Zenodo. 10.5281/zenodo.22104155

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