

One Table, Two Correct Answers

— Every Subgroup Says One Thing and the Pooled Data Says the Other: the Separator Is Not in the Table —

[Paper 183]

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— *From thinking to syncing.* —

ABSTRACT

Treatment **A** wins in every subgroup and loses once the groups are pooled. This reversal is usually told as a statistical curiosity. This paper writes down that three different things travel under the one name. **NO NEW MATHEMATICAL THEOREM AND NO NEW STATISTICAL METHOD IS CLAIMED.** First, it is arithmetic, not statistics. From $a/b < c/d$ and $e/f < g/h$ it does not follow that $(a + e)/(b + f) < (c + g)/(d + h)$. The smallest non-degenerate counterexample has denominators summing to 19: $1/3 < 2/5$ and $5/7 < 3/4$, yet $3/5 > 5/9$ (Section 2). Second, the mechanism is a difference of weights. In a table constructed here, **A** wins in both groups yet loses on pooling by 0.54 to 0.76 — with exactly 100 cases in each arm, so only the mixture differs (Section 3). Third, the fence is balanced allocation. Searching all 669124 tables with equal group sizes in both arms produced not one reversal — unbalancing them brings back 1166 (Section 4). Fourth, and this is the core. One table carries two correct answers. Read the group as a confounder and the adjusted averages are 0.70 against 0.60, so **A** wins; read it as a mediator and the pooled averages are 0.54 against 0.76, so **B** wins — with not a single number changed (Section 5). Fifth, what must move for the answer to move is an arrow, not a number, and an arrow is not in the table (Section 6).

Scope of this paper: NO NEW MATHEMATICAL THEOREM AND NO NEW STATISTICAL METHOD IS CLAIMED — Yule, Simpson, Blyth, and the rules for when adjustment is licensed by a causal diagram are all standard. The table was constructed for this paper and is not real data — no medical or social claim of any kind is made. On Paper 124: that paper's “aggregation” is the licence to replace distant sources by a few quantities in an N -body calculation, not the pooling of subgroup statistics — the same word for a different thing. On Paper 154: the pattern that two operations fail to commute is that paper's, and no novelty is claimed for it here. What is added is that here which order is correct is neither taste nor convention: it is settled by a causal fact the data does not contain. What is added is separating the arithmetic, the mechanism and the causal question, finding the smallest non-degenerate counterexample by exhaustive search, confirming by exhaustive search that balanced allocation makes reversal impossible, and putting two causal structures on one table and getting opposite answers in exact fractions.

1. Notation and position

Write A and B for two treatments and split the cases into two groups G_1 and G_2 . The success rate is successes over attempts. All arithmetic is in exact fractions and all searches are exhaustive.

Position: Paper 164 wrote that “probability” has four addresses. This paper stands beside it and exhibits something the data does not settle — one table yielding two correct answers.

2. It is arithmetic, not statistics

Observation 2.1

For fractions, $a/b < c/d$ together with $e/f < g/h$ does not give $(a + e)/(b + f) < (c + g)/(d + h)$. No probability, no sample, no model is involved — it is a fact about how fractions add.

Verification 2.2: Restricting to fractions strictly between 0 and 1 and searching exhaustively, the counterexample with the smallest sum of denominators is this.

	side A	side B	direction
group G_1	1/3	2/5	$A < B$
group G_2	5/7	3/4	$A < B$
pooled	3/5	5/9	$A > B$

The denominators sum to **19**. (QED)

Observation 2.3

So saying that “statistics is counterintuitive” misses the mark here. What is being violated is an intuition about fractions, and statistics has not yet begun.

3. The mechanism: pooling is a weighted average with different weights

Verification 3.1: The following table is constructed here. It is not real data.

	group G_1	group G_2	pooled
treatment A	$9/10 = 9/10$	$45/90 = 1/2$	$54/100 = 27/50$
treatment B	$72/90 = 4/5$	$4/10 = 2/5$	$76/100 = 19/25$
direction	$A > B$	$A > B$	$A < B$

Each treatment has exactly 100 cases, so nothing here turns on one arm being larger. (QED)

Verification 3.2: What differs is the mixture.

	share in G_1	share in G_2
treatment A	1/10	9/10
treatment B	9/10	1/10

(QED)

Observation 3.3

The pooled number is each arm's average over its own mixture. It answers “what happens to a case of the kind that ends up in this arm” — which is not the question being asked.

4. The fence is balanced allocation

Observation 4.1

If both arms have the same group sizes, pooling is the **same weighted average** on both sides. **An average with the same weights preserves order, so no reversal is possible.**

Verification 4.2: Letting group sizes run to **12** and searching all balanced tables exhaustively.

allocation	tables examined	reversals
balanced (same sizes in both arms)	669124	0
unbalanced	512120	1166

(QED)

Observation 4.3

Not one case among the balanced tables. This is what fixing the group sizes buys — not making reversal unlikely but making it logically impossible.

Observation 4.4

But this fence can only be built for groups one knows about. For groups one does not know, neither balancing the allocation nor adjusting afterwards is available.

5. The core: one table, two correct answers

Observation 5.1

Take the table of Section 3 unchanged. **Not a single number is altered.** Across everyone, G_1 is half and G_2 is half.

Verification 5.2: First reading — the group is a confounder (it affects both which treatment a case receives and how it turns out). Then the effect is the average weighted by each group's share of the **whole population**.

treatment	adjusted average	verdict
<i>A</i>	$7/10 = 0.7000$	
<i>B</i>	$3/5 = 0.6000$	A is better

(QED)

Verification 5.3: Second reading — the group is a mediator (the treatment itself moves cases between groups, and that movement is part of how it works). Adjusting would subtract the treatment's own effect, so the pooled number is the effect.

treatment	pooled average	verdict
<i>A</i>	$27/50 = 0.5400$	
<i>B</i>	$19/25 = 0.7600$	B is better

(QED)

Observation 5.4

Opposite answers from the same numbers. The table is identical in both cases, so no amount of studying it decides which reading is right.

6. What must move is an arrow, not a number

Observation 6.1

causal structure	what to compute	answer
group $\rightarrow$ treatment (confounder)	adjusted average	<i>A</i>
treatment $\rightarrow$ group (mediator)	pooled average	<i>B</i>
treatment $\rightarrow$ group $\leftarrow$ outcome (collider)	pooled, do not condition	<i>B</i>

Observation 6.2

The mediator and the collider share an answer for opposite reasons. With a mediator, adjusting removes a real effect; with a collider, adjusting creates an association that is not there. Three structures and only two computations.

Observation 6.3

The separator is the direction of an arrow, and an arrow is not an entry in the table. When Paper 95 separated convention from fact it wrote that a convention must be a number and not an arrangement. Here the other side is needed — what settles the answer is not a number but a structure outside the numbers.

7. The census

Observation 7.1

layer	what it is about	settled?
arithmetic	how fractions add	settled (Ver. 2.2)
mechanism	a difference of weights	settled (Ver. 3.2)
causal	which one to compute	not settled by the table

Observation 7.2

The pattern that two operations fail to commute belongs to Paper 154 (a limit needs its order). No novelty is claimed for it here. What is added is how the correct order gets decided — in 154 the question fixes the order, while here it is fixed by a causal fact that the data does not contain.

8. Conclusion

1. The reversal is arithmetic (Ver. 2.2): smallest denominators sum to 19, with $3/5 > 5/9$.
2. The mechanism is a difference of weights (Ver. 3.2): 100 cases in each arm, mixtures 1/10 against 9/10.
3. Balanced allocation makes reversal impossible (Ver. 4.2): 0 among 669124.
4. Unbalancing brings it back (Ver. 4.2): 1166 cases.
5. One table carries two correct answers (Ver. 5.2, 5.3): 0.70 against 0.60 says *A*; 0.54 against 0.76 says *B*.
6. Three structures, two computations (Obs. 6.2).

Closing: Whether to look group by group or to pool is not the kind of question that careful reading of the data settles. Which average is the answer depends on whether the group sits before the treatment or after it, and that is not written in any cell of the table. It is not that there are too few numbers; it is that numbers are not enough.

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