

What Classifies a Defect Is the Target Space, Not the Equation

— Homotopy Does the Counting, and “Stable” Is Two Words — [Paper 185]

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— *From thinking to syncing.* —

ABSTRACT

This corpus has handled topology many times (Papers 67, 68, 114, 125, 128, 146, 151). It has never asked what classifies a defect. The answer is a homotopy group of the space the field takes values in — not of the space it lives on, and not of the equations. **NO NEW MATHEMATICAL THEOREM AND NO NEW PHYSICAL LAW IS CLAIMED.** First, winding numbers are whole numbers. Adding a $\sin 3\theta$ wobble to the angle left the winding at 1.0, unmoved (Section 2). Second, and this is the core. On the same plane in the same dimension, changing the target changes which defects are allowed. A half-turn defect is forbidden for a field of arrows and allowed for a field of undirected lines — with no equation and no plane altered (Section 3). Third, two halves return to the start. A half-defect is a path to the antipode on the sphere, and two of them landed back at the starting point — the classifying group has order 2 (Section 4). Fourth, when the group is non-abelian a defect's charge changes on being carried around another. Using Q_8 , the group Paper 152 built, an i carried around a j came back as $-i$ (Section 5). Fifth, and this is the brake. “Stable” is one word for two things. In one dimension the energy has a genuine minimum at scale $\lambda = 1$; from two dimensions up there is no minimum at all and squeezing always pays — a topologically non-trivial configuration can still be driven to a point (Section 6).

Scope of this paper: **NO NEW MATHEMATICAL THEOREM AND NO NEW PHYSICAL LAW IS CLAIMED** — the homotopy classification of defects (Toulouse–Kléman, Mermin) and Derrick's scaling argument are standard. Nothing is claimed about any particular material or experiment. On Paper 128: that paper treated vector fields on the base manifold and tied the index sum to the Euler characteristic. What is looked at here is the target space, a different space — one word “topological” covering two of them. On Papers 146 and 151: those computed one defect, the Dirac monopole, by covers and Čech cochains. This paper says which general rule that one is an instance of. What is added is showing in numbers that changing only the target changes which defects are allowed, verifying the composition of half-defects as paths on the sphere, exhibiting in Q_8 how a charge moves by conjugation, and splitting the two senses of “stable” with Derrick's scaling.

1. Notation and position

A field assigns to each point of the base space a value in the **target space** (the order parameter space). The homotopy class of the map from a small sphere surrounding a defect into the target space is the defect's **class**. Every number here was computed for this paper.

Position: Paper 128 treated Poincaré–Hopf and counted the indices of the zeros of a vector field on the base. What is counted here are the holes in the target. The same word “topological” is being used about two different spaces.

2. Winding numbers are whole numbers

Verification 2.1: For a field of unit vectors along a circle, measure the total turning over one loop.

angle of the field	measured winding	whole?
θ	1.0	yes
$-\theta$	-1.0	yes
2θ	2.0	yes
$\theta + \sin 3\theta$	1.0	yes
constant	0.0	yes

(QED)

Observation 2.2

The wobble does not move the number. That is what a topological invariant is. And there is one reason it is a whole number — after one loop the field must come back to itself.

3. The core: same plane, different target, different defects

Observation 3.1

For a field of undirected lines, n and $-n$ are the same state. So the target is not the circle but the circle with antipodes identified, and after one loop the field need only return to itself or its opposite.

Verification 3.2

configuration	turning	field of arrows	field of lines
$k = 1$	1.0	allowed	allowed
$k = 2$	2.0	allowed	allowed
$k = 1/2$	0.5	forbidden	allowed
$k = -1/2$	-0.5	forbidden	allowed
$k = 3/2$	1.5	forbidden	allowed

In the half-turn the angle moves by π , which is 0 modulo π (measured: 0.0). (QED)

Observation 3.3

The half-turn defect is forbidden to arrows and allowed to lines. The plane is the same, the dimension is the same, and not one equation has been written. The only thing changed is the space the field takes values in. That is what does the classifying.

4. Two halves return to the start

Observation 4.1

For a field of lines in three dimensions the target is the sphere with antipodes identified. A half-defect is a path to the antipode; a whole one is a path back to the same point.

Verification 4.2: Rotate the north pole about the $\mathfrak{x}$ axis by half-turns and look at the end point.

half-defects	end point	class
1	the antipode	non-trivial
2	the starting point	trivial
3	the antipode	non-trivial

(QED)

Observation 4.3

Two halves come back to the start — the group of classes has order two, and a whole-number defect can be deformed away. “There is a defect of winding one” can, depending on the target, mean the same as nothing.

5. Non-abelian: the charge changes on a round trip

Observation 5.1

When the classifying group is non-abelian, a defect's charge is defined **only up to conjugation**. Take Q_8 , the group built in Paper 152, as the defect group.

Verification 5.2

computation	result
ij	k
ji	$-k$
jij^{-1}	$-i$

(QED)

Observation 5.3

An i carried once around a j came back as $-i$. Nothing was done to the defect itself — it was only carried around another. So “this defect has charge i ” means nothing until a path is named.

Verification 5.4: The conjugacy classes are $\{1\}$, $\{-1\}$, $\{\pm i\}$, $\{\pm j\}$ and $\{\pm k\}$. (QED)

Observation 5.5

The two-element classes are exactly the defects whose sign is not an invariant. Paper 152 showed that Q_8 and D_4 share a character table without being isomorphic. Here the same Q_8 appears in a completely different role — one group, two uses.

6. The brake: “stable” is one word for two things

Observation 6.1

Topologically stable means it cannot be removed by continuous deformation. It does not mean it sits at a minimum of the energy.

Verification 6.2: Write the energy of a static field as $E = E_{\text{grad}} + E_{\text{pot}}$ and replace $f(x)$ by $f(\lambda x)$. Larger λ means squeezed. Then $E(\lambda) = \lambda^{2-d}E_{\text{grad}} + \lambda^{-d}E_{\text{pot}}$. Taking both pieces as 1:

d	$\lambda = 1$	$\lambda = 1000$	where the minimum is
1	2.0	1000.0	a true minimum at $\lambda = 1$
2	2.0	1.0	none: squeezing always pays
3	2.0	0.001	none: it collapses to a point
4	2.0	10^{-6}	none

Searching a wide grid of scales, $d = 1$ has an interior minimum at $\lambda = 1$ with $E = 2.0$, while $d = 2$ and $d = 3$ were pushed to the edge of the grid. (QED)

Observation 6.3

From two dimensions up there is no size the energy prefers. Squeezing always pays. So a configuration can be topologically non-trivial and still be driven to a point. Topology forbids undoing it, not shrinking it.

Observation 6.4

This is why a real defect needs something more — a gauge field, a higher-derivative term, or a conserved quantity. And that something does not come out of the homotopy group.

7. The census

Observation 7.1

group	defect	codimension	what the target needs
π_0	wall	1	to be disconnected
π_1	line (vortex, disclination)	2	a hole
π_2	point (monopole)	3	a two-dimensional hole
π_3	texture	4	a three-dimensional hole

Observation 7.2

The group says only which classes exist. It does not say that a defect exists, that one is stable, or what it costs. Paper 163 wrote that existence and construction are different sentences — what is added here is classification as a third. The three run in order of strength: classification lists the classes, existence says a class is not empty, construction hands one over.

Observation 7.3

And this paper's fence is the target space, following linearity in Paper 171, integrality in 173, compactness in 177 and the number of options in 184. Each marks a place where moving it changes the answer.

8. Conclusion

1. A wobble does not move the winding (Ver. 2.1): still 1.0 with $\sin 3\theta$ added.
2. Changing the target changes which defects are allowed (Ver. 3.2): the half-turn is forbidden to arrows and allowed to lines.
3. Two halves are trivial (Ver. 4.2): the group has order 2.
4. A non-abelian charge changes on a round trip (Ver. 5.2): $jij^{-1} = -i$.
5. “Stable” is two words (Ver. 6.2): no energy minimum from $d \geq 2$.
6. Classification is neither existence nor construction (Obs. 7.2).

Closing: When something is called topologically protected, the next question is the topology of which space. Not the space the field lives on but the space its values live in. And what is protected is only that it cannot be undone — not that it cannot be crushed.

References

1. Derrick, G. H. (1964). *Comments on Nonlinear Wave Equations as Models for Elementary Particles*. Journal of Mathematical Physics, 5(9).
2. Toulouse, G., Kléman, M. (1976). *Principles of a Classification of Defects in Ordered Media*. Journal de Physique Lettres, 37(6).
3. Volovik, G. E., Mineev, V. P. (1977). *Investigation of Singularities in Superfluid He-3 and Liquid Crystals by Homotopic Topology Methods*. Soviet Physics JETP, 45(6).
4. Mermin, N. D. (1979). *The Topological Theory of Defects in Ordered Media*. Reviews of Modern Physics, 51(3).
5. Kibble, T. W. B. (1976). *Topology of Cosmic Domains and Strings*. Journal of Physics A, 9(8).
6. Yamagishi, Y. (2026). *The Euler-Characteristic Origin of the 4π Patch* [Paper 67]. Zenodo. 10.5281/zenodo.21642170
7. Yamagishi, Y. (2026). *Which Objects Know They Are in Four Dimensions?* [Paper 114]. Zenodo. 10.5281/zenodo.21963544
8. Yamagishi, Y. (2026). *There Are Three Hopfs* [Paper 125]. Zenodo. 10.5281/zenodo.22079384
9. Yamagishi, Y. (2026). *$4\pi = 2\pi\chi$ Holds in Three Dimensions and Nowhere Else* [Paper 128]. Zenodo. 10.5281/zenodo.22079576
10. Yamagishi, Y. (2026). *A Vortex Line Cannot End Because of an Identity, Not a Law* [Paper 130]. Zenodo. 10.5281/zenodo.22079996

11. Yamagishi, Y. (2026). *The Dirac String Is the Mark of Trying to Write It on One Sheet* [Paper 146]. Zenodo. 10.5281/zenodo.22082868
12. Yamagishi, Y. (2026). *A Two-Set Cover Returns the Wrong Answer for the Circle* [Paper 151]. Zenodo. 10.5281/zenodo.22084096
13. Yamagishi, Y. (2026). *The Same Character Table, and Still Not Isomorphic* [Paper 152]. Zenodo. 10.5281/zenodo.22084202
14. Yamagishi, Y. (2026). *“A Good Code Exists” and “Here Is a Good Code” Are Different Sentences* [Paper 163]. Zenodo. 10.5281/zenodo.22092677
15. Yamagishi, Y. (2026). *Kodaira's Fence Is Integrality, Not Positivity* [Paper 173]. Zenodo. 10.5281/zenodo.22094507
16. Yamagishi, Y. (2026). *“The Curse of Dimensionality” Is Not One Curse* [Paper 182]. Zenodo. 10.5281/zenodo.22104170

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