

The Gaussian Is Special for More Than One Reason

— Change What You Hold Fixed and a Different Law Wins, and Uncorrelated Is Not Independent — [Paper 187]

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— *From thinking to syncing.* —

ABSTRACT

Asked why the gaussian turns up everywhere, one usually answers with the central limit theorem alone. This paper writes down four unrelated reasons, each holding a different thing fixed. **NO NEW MATHEMATICAL THEOREM IS CLAIMED.** First, finite variance is a condition and not a decoration. Averaging a thousand normal draws shrinks the spread from 1.345 to 0.0427; averaging a thousand Cauchy draws moves it from 2.003 to 2.017, which is nothing at all — the quartiles matched those of a single draw (Section 2). Second, stability alone does not pick the gaussian. Both the normal and the Cauchy are stable, with relative gaps under 0.004 — stability is a family, not a point (Section 3). Third, maximum entropy depends on what is held fixed. Fix the variance and the gaussian wins (measured 1.418 against a theoretical ceiling of 1.419); fix the mean and the exponential wins; fix the range and the uniform does (Section 4). Fourth, and this is the core. Rotational invariance together with independent coordinates forces the gaussian — turning the product density by forty-five degrees changed it by 5.6×10^{-17} for the gaussian alone. No sum and no limit appears anywhere (Section 5). Fifth, the first attempt to check a fourth characterisation failed. Measuring the correlation of the sample mean with the sample variance gave zero for the normal, the uniform and the Laplace alike. Every symmetric law does that — correlating with the square of the mean instead gives +0.0006 for the gaussian against −0.300 and +0.299 (Section 6).

Scope of this paper: **NO NEW MATHEMATICAL THEOREM IS CLAIMED** — the central limit theorem, stable laws, maximum entropy, the Herschel–Maxwell argument, and the characterisation by independence of the sample mean and variance are all standard. No distribution is judged “correct”. On Paper 139: that paper treated the rate at which empirical measures converge. The question here is why the limit is gaussian, which is a different one. What is added is setting four reasons side by side by what each holds fixed, confirming with quartiles that averaging Cauchy variables does nothing, measuring the force of rotational invariance to machine precision, and extracting from a failed test of my own the fact that uncorrelated and independent are different.

1. Notation and position

Everything sampled was sampled for this paper, with sample sizes stated. Laws are centred at zero and spread is measured by quantiles, since the Cauchy has no variance.

Position: Paper 164 wrote that “probability” has four addresses. This paper stands beside it with one distribution reached by four doors — a case where it is the object rather than the word that has several entrances.

2. Finite variance is a condition, not a decoration

Verification 2.1: Over two hundred thousand trials, the interquartile range of the mean of n draws.

n	normal: spread of the mean	Cauchy: the same
1	1.345	2.003
10	0.425	1.995
100	0.135	2.003
1000	0.0427	2.017

(QED)

Verification 2.2: For the Cauchy, compare a single draw with the mean of a thousand.

quantile	one draw	mean of a thousand
25%	−1.003	−1.000
50%	−0.0006	−0.003
75%	1.004	0.996

(QED)

Observation 2.3

The normal mean shrinks like one over the square root of n and the Cauchy mean does nothing — a thousand of them average to the same law as one. “Collect enough and the average settles” is true only when the variance is finite.

3. Stability alone does not pick the gaussian

Observation 3.1

A law is **stable** when a sum of independent copies is the same law up to shift and scale.

Verification 3.2: Interquartile ranges over a million trials.

law	k	scale	relative gap
normal	2	$\sqrt{k}$	0.00075
normal	100	$\sqrt{k}$	0.00060
Cauchy	2	k	0.00213
Cauchy	100	k	0.00244

(QED)

Observation 3.3

Both are stable, with different exponents — the square root of k for the normal and k for the Cauchy. Stability is a family and not a point, and what picks the gaussian out of it is the extra condition of finite variance.

4. Maximum entropy: the answer is decided by what is held fixed

Verification 4.1: Differential entropy of laws with variance one, from histograms of two million points.

law	variance	entropy
normal	0.999	1.4184
Laplace	1.001	1.3468
uniform	1.001	1.2423
two-point	1.000	−3.929

The theoretical ceiling at variance one is **1.4189**. (QED)

Observation 4.2

The gaussian wins — and only because the variance was what was fixed.

what is held fixed	the winner
the variance	normal
the mean, on the positive half-line	exponential
the range	uniform

Observation 4.3

“The gaussian is the least informative distribution” is a half-sentence. The full one is “least informative given that the variance is known”, and changing what is known changes the answer.

5. The core: symmetry alone forces the gaussian

Observation 5.1

The argument of Herschel and Maxwell. **If the two coordinates of a point in the plane are independent and the density depends only on the distance from the origin, the law must be gaussian. No sum and no limit appears.**

Verification 5.2: Turn the product density $f(x)f(y)$ by forty-five degrees and measure the largest change, over a grid of forty-one by forty-one.

law	largest change under the turn
normal	5.6×10^{-17}
Cauchy	9.1×10^{-3}
Laplace	6.3×10^{-2}
uniform	8.3×10^{-2}

(QED)

Observation 5.3

Only the gaussian is unchanged, and it is unchanged to machine precision. What selects it here is symmetry — nothing was added up and no limit was taken. What is being demanded is quite unlike Section 2.

6. Uncorrelated is not independent

Observation 6.1

There is a fourth characterisation — the gaussian is the only law for which the sample mean and the sample variance are independent. The first attempt to check it failed.

Verification 6.2: Four hundred thousand samples of size eight, measuring the correlation.

law	correlation of mean and variance
normal	−0.003
uniform	−0.0007
Laplace	+0.003
exponential	+0.694

The uniform and the Laplace read as zero, exactly like the normal. (QED)

Observation 6.3

This does not refute the characterisation — the instrument was wrong. For a symmetric law the third central moment governing this correlation vanishes, so all of them read zero. Being uncorrelated and being independent are different statements.

Verification 6.4: So correlate the **square** of the mean with the variance, which independence would also force to zero.

law	correlation of the squared mean with the variance
normal	+0.0006
uniform	−0.300
Laplace	+0.299
exponential	+0.714

(QED)

Verification 6.5: Look directly: the spread of the variance among samples whose mean was small against those whose mean was large.

law	ratio of the two spreads
normal	1.002
uniform	0.935
Laplace	1.496
exponential	7.482

(QED)

Observation 6.6

Only the gaussian stays at zero. The uniform and the Laplace were uncorrelated and were not independent. And this characterisation touches neither sums, nor entropy, nor symmetry — it is a fourth and separate reason.

7. The census

Observation 7.1

why the gaussian	what is held fixed	what happens without it
a limit of sums	finite variance	another stable law, such as Cauchy
stability alone	nothing	a whole family survives
maximum entropy	the variance	mean gives exponential, range gives uniform
rotation and independence	symmetry only	no sum and no limit needed
mean and variance independent	a property of samples	unrelated to the others

Observation 7.2

These five are not a staircase. The limit of sums is a statement about a process, maximum entropy about knowledge, rotational invariance about symmetry, and independence about samples. One distribution with four quite different entrances.

Observation 7.3

So “assume a gaussian” does not name one assumption. Was finite variance assumed, or was it said that only the variance is known, or was symmetry demanded? Each fails in a different place.

8. Conclusion

1. Averaging does nothing to a Cauchy (Ver. 2.1): **2.003 to 2.017**.
2. Stability is a family (Ver. 3.2): **the Cauchy is stable too, scaling by k** .
3. Maximum entropy is decided by what is fixed (Ver. 4.1): **measured 1.4184 against a ceiling of 1.4189**.
4. Symmetry alone forces the gaussian (Ver. 5.2): **a change of 5.6×10^{-17} under rotation**.
5. Uncorrelated is not independent (Ver. 6.2, 6.4): **all correlations zero, yet -0.300 and $+0.299$ against the squared mean**.

Closing: There is no single answer to why the gaussian is everywhere. Was something summed, was the variance the only thing known, was symmetry imposed? There are four doors, each holding a different thing fixed — and when the thing held fixed is removed, the law that appears and the way it breaks differ in all four.

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