

“Unprovable” Is Not a Property of a Sentence

— One Sentence Becomes Independent, Provable or Refutable Depending Only on Which Axioms Stand —

[Paper 189]

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— *From thinking to syncing.* —

ABSTRACT

This corpus has two papers on failure — Paper 25 split “unsolvable” three ways by which tools are allowed, and Paper 179 split “cannot be constructed” three ways by how far a search could get. Both were about objects. This paper does the same for sentences. **NO NEW MATHEMATICAL THEOREM IS CLAIMED.** First, an undecidable sentence is not a mysterious one. Under the theory that a relation is reflexive and transitive, the sentence that it is symmetric holds in 5 of the 29 models on three elements and fails in the other 24 — independence is shown by exhibiting both (Section 2). Second, and this is the core. The same sentence takes three different statuses when axioms are added. Adding symmetry leaves 5 models and the sentence holds in all of them, so it is a theorem; adding that some pair runs one way leaves 24 models and it fails in all of them, so it is refuted; adding nothing leaves it independent (Section 3). Third, the fence is consistency. A theory demanding both reflexivity and irreflexivity has 0 models, and there everything is “provable” — incompleteness is what a consistent theory pays (Section 4). Fourth, four things that do not follow. Undecidable does not mean nobody can know, nor that there is no truth of the matter, nor that the theory is wrong (Section 5). Fifth, this paper does not prove Gödel's theorem. What is shown is only that independence belongs to a pair; the statement about theories containing arithmetic is quoted (Section 5).

Scope of this paper: NO NEW MATHEMATICAL THEOREM IS CLAIMED — the completeness theorem, the incompleteness theorems, and the independence of the parallel postulate and of the continuum hypothesis are all standard. And this paper does not prove the incompleteness theorem — exhaustive enumeration of finite models reaches only as far as showing that independence belongs to the pair of a theory and a sentence. On Papers 25 and 179: their separators were which tools are allowed and how far a search could get, and both were about objects. The separator here is under which axioms, and it is about sentences — the three make one row. What is added is placing a third kind of “cannot” in the corpus, showing independence by enumerating models rather than by any paradox, putting the independent, provable and refutable statuses of one sentence in a single table, and locating the fence at consistency.

1. Notation and position

Consider a binary relation R on a finite set. A theory is a collection of conditions imposed on R , and a **model** is an R meeting them. A sentence S is **independent** of a theory T when there are models of T in which S holds and models in which it fails. Every enumeration below is exhaustive.

Position: Paper 163 separated existence from construction and Paper 188 inserted choice between them. This paper is earlier still — about whether the thing can be said at all.

2. An undecidable sentence is not a mysterious one

Observation 2.1

Let T say that R is reflexive and transitive, and let S say that R is symmetric. Count every model of T on k elements and how many satisfy S .

Verification 2.2

k	models of T	S holds	S fails	verdict
1	1	1	0	T decides S
2	4	2	2	S is independent
3	29	5	24	S is independent

(QED)

Observation 2.3

From two elements on there are models where S holds and models where it fails. So T can neither prove nor refute S — and that was shown by exhibiting both, with no self-reference and no paradox.

Observation 2.4

At $k = 1$ it is not independent — on one element reflexivity and transitivity force symmetry. Whether a sentence is independent depends not only on the theory and the sentence but on how large a model is allowed.

3. The core: one sentence, three statuses

Verification 3.1: On three elements, add one axiom at a time to the base theory.

theory	models	S holds	S fails	status of S
reflexive and transitive	29	5	24	independent
and symmetric	5	5	0	a theorem
and total	13	1	12	independent
and some pair runs one way	24	0	24	refuted

(QED)

Observation 3.2

Not one character of the sentence changed. What changed is only which axioms are standing. That alone makes the same sentence independent, provable or refutable.

Observation 3.3

So “this sentence cannot be proved” means nothing on its own. Independence is not a property of a sentence but a property of the pair of a sentence and a set of axioms. It has the shape of Paper 185’s reply to “topologically protected”: the topology of which space.

Observation 3.4

Adding axioms shrinks the models, from 29 down to 5, 13 and 24. Matters get settled because the allowed worlds were narrowed, not because anything was discovered.

4. The fence is consistency

Verification 4.1: Count the models of a theory demanding that R be both reflexive and irreflexive.

theory	models
reflexive and irreflexive	0

(QED)

Observation 4.2

With no models at all, every sentence holds in all of them — and so does its negation. A contradictory theory is complete, and useless.

Observation 4.3

So incompleteness is not a defect of a theory but the price of consistency. It has the shape of Paper 184, where a list of desirable conditions could not all hold, and the question there was also what to give up — completeness, or consistency.

5. What does not follow

Observation 5.1

what is often said	in fact
undecidable means nobody can know	false: a larger theory settles it
undecidable means there is no truth	false: true in some models, false in others
undecidable means the theory is wrong	false: it is the price of being consistent
adding axioms fixes it once	false: the new theory has new gaps

Observation 5.2

The last row is the actual content of the incompleteness theorem — any consistent theory containing arithmetic whose axioms can be listed mechanically leaves a new gap.

Observation 5.3

This paper does not prove that. Exhaustive enumeration of finite models reaches only the single point that independence belongs to a pair. The last row is quoted from Gödel (1931) and does not come out of the computation here.

6. The census: three kinds of “cannot”

Observation 6.1

word	about what	separator	move it and
unsolvable (Paper 25)	objects	which tools are allowed	more tools, it is solved
not constructible (Paper 179)	objects	how far a search gets	the reach of the search
unprovable (this paper)	sentences	under which axioms	add axioms, it is settled

Observation 6.2

The three are not a staircase. Paper 25 is about permission, Paper 179 about search, and this one about context. One word “cannot” covering three things that differ in what must be moved.

Observation 6.3

And each calls for a different next question — “which tools were allowed”, “how far was it searched”, “under which axioms”.

7. Conclusion

1. Independence can be shown by exhibiting both kinds of model (Ver. 2.2): **5 against 24 on three elements.**
2. At one element it is not independent (Obs. 2.4): **size matters too.**
3. One sentence, three statuses (Ver. 3.1): **5/24, 5/0, 0/24.**
4. It gets settled by narrowing the worlds (Obs. 3.4): **the models drop from 29.**
5. The fence is consistency (Ver. 4.1): **a contradictory theory has 0 models.**
6. This paper does not prove the incompleteness theorem (Obs. 5.3).

Closing: When a proposition is said to be unprovable, the next question is under which axioms. A sentence taken by itself has no such status — being independent is not a fact about the sentence but a fact about its relation to the axioms that happen to be standing.

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