

The Thing That Smooths Is What Makes the Pattern

— Diffusion Breaks the Symmetry, and the Fence Is the Ratio of Two Diffusion Rates — [Paper 191]

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— *From thinking to syncing.* —

ABSTRACT

Out of equations containing no length at all comes a pattern with a definite one. And what brings it about is diffusion, which ordinarily smooths everything away. This paper follows that reversal in numbers and says where the length came from. **NO NEW MATHEMATICAL THEOREM AND NO NEW PHYSICAL LAW IS CLAIMED.** First, neither ingredient makes anything alone. With reaction only, the uniform state is stable (trace -0.4 , determinant 0.23 , growth rate -0.2 at zero wavenumber); with diffusion only, every wavenumber decays (Section 2). Second, and this is the core. Put them together and give the two rates different speeds, and the uniform state goes unstable. At a ratio of 1 every wavenumber still decays; at 5 a band grows at rate 0.127 , and at 30 at 0.529 (Section 3). Third, the fence is a threshold in the ratio. At 3.4135 the behaviour switches: below it nothing happens at any wavelength, above it a band opens — and changing d_1 through $1, 2$ and 4 left the threshold ratio at 3.4135 (Section 4). Fourth, the length is built from the coefficients. The wavenumber at threshold obeys $k^4 = \det J / (d_1 d_2)$ to 10^{-5} — the length is a diffusion constant divided by a rate, and the container never appears (Section 5). Fifth, I had the direction backwards. The wavelength grows with the ratio, from 13.62 to 22.10 , where a first draft of this section said it shrinks (Section 5).

Scope of this paper: NO NEW MATHEMATICAL THEOREM AND NO NEW PHYSICAL LAW IS CLAIMED — the Turing instability, the dispersion relation and the threshold condition are all standard. Nothing is claimed about biology — whether the markings of any actual animal arise by this mechanism is not a question this paper touches. What is treated is only the dispersion relation of a linearised two-species system. On Paper 175: that paper split “symmetry breaking” into the explicit, the spontaneous and the anomalous, and wrote that spontaneous breaking does not occur in a finite system. That was a statement about thermodynamic phase transitions, whereas the breaking here is a linear instability of a deterministic equation — one phrase over two matters, and this one does occur in a finite domain. What is added is setting side by side in numbers the fact that two ingredients which do nothing alone do the opposite together, confirming across three values of d_1 that the threshold depends on the ratio alone, checking a closed form to 10^{-5} to locate where the length comes from, and separating what the container fixes from what the chemistry fixes.

1. Notation and position

Linearise a two-species reaction-diffusion system about a uniform steady state. With J the reaction jacobian and d_1, d_2 the diffusion constants, a disturbance of wavenumber k grows at the largest real part of the eigenvalues of $J - k^2 \text{diag}(d_1, d_2)$, and that number is called the **growth rate** below. Every number was computed for this paper.

Position: Paper 117 split scale invariance into four roots and Paper 120 wrote the two corridors along which dimensional analysis fails. This paper stands beside them with the case where a length appears where there was none.

2. Neither ingredient makes anything alone

Verification 2.1: Take the reaction jacobian J with first row **0.9** and **−1.0** and second row **1.4** and **−1.3**, of trace **−0.4** and determinant **0.23**.

quantity	value	meaning
trace	−0.4	negative
determinant	0.23	positive
growth rate at $k = 0$	−0.2	the uniform state is stable

Without diffusion every disturbance dies. (QED)

Verification 2.2: With diffusion only, the growth rate is $-dk^2$.

k	growth rate	behaviour
0.5	-0.25	decays
1.0	-1.0	decays
4.0	-16.0	decays

(QED)

Observation 2.3

One of them protects the uniform state and the other flattens every bump. Neither looks like an ingredient for making a pattern.

3. The core: together, with different speeds, the uniform state breaks

Verification 3.1: The same reaction with two diffusion constants.

d_1	d_2	largest growth rate	fastest wavenumber
1	1	-0.2	none, all decay
1	5	0.127124	0.4940
1	10	0.312965	0.4610
1	30	0.528914	0.3940

(QED)

Observation 3.2

With the two speeds equal, every wavenumber still decays. Make one of them faster and a band of wavenumbers grows. The thing that smooths has made the pattern.

Observation 3.3

And what does the work is not the speed but the difference — speeding both up equally changes nothing if the ratio is unchanged. The next section checks that.

4. The fence is a threshold in the ratio

Verification 4.1: Walk the ratio up and bisect for the point where growth becomes possible.

ratio	largest growth rate	verdict
3.0721	-4.0×10^{-2}	no pattern
3.3793	-3.7×10^{-3}	no pattern
3.4476	$+3.6 \times 10^{-3}$	pattern
3.7548	$+3.4 \times 10^{-2}$	pattern

The measured threshold is **3.413460**. (QED)

Verification 4.2: Changing d_1 to see whether the threshold really depends on the ratio.

d_1	threshold d_2	ratio
1	3.413460	3.41346
2	6.826917	3.41346
4	13.653882	3.41347

(QED)

Observation 4.3

The threshold depends on the ratio alone, agreeing to six digits. And it is sharp — below it nothing happens at any wavelength, above it a band opens. There is no gradual onset.

5. The length is built from the coefficients

Observation 5.1

At threshold the growing band shrinks to a single point, and that wavenumber has a closed form:

$$k^4 = \det J / (d_1 d_2).$$

Verification 5.2

d_1	measured k	$(\det J / (d_1 d_2))^{1/4}$	difference
1	0.509500	0.509487	1.3×10^{-5}
2	0.360300	0.360262	3.8×10^{-5}
4	0.254700	0.254743	4.3×10^{-5}

(QED)

Observation 5.3

Read the formula as a statement about units. The determinant carries the units of a rate squared and the product of the diffusion constants those of a length to the fourth over a time squared, so k^4 comes out as one over a length to the fourth. The length is a diffusion constant divided by a rate and nothing else. The container appears nowhere in it.

Verification 5.4: The fastest-growing wavelength as the ratio rises.

d_2/d_1	fastest k	wavelength
10	0.46131	13.62044
40	0.37450	16.77732
160	0.28430	22.10027

(QED)

Observation 5.5

The wavelength grows with the ratio. A first draft of this section said it shrinks, and the measurement says otherwise. Letting the inhibitor reach further makes the stripes wider — which is the sensible direction after all.

Verification 5.6: What the container fixes is not the length but the count. At a ratio of **10** the selected wavelength is **13.6295**, and

length of the container	stripes that fit
30	2.20
60	4.40
120	8.80

(QED)

Observation 5.7

The scale is predicted by the chemistry and the count is decided by the vessel. “How big the pattern is” and “how many there are” come from different places.

6. The census

Observation 6.1

ingredients	what happens	result
reaction only	no wavenumber of its own	the uniform state is stable
diffusion only	it flattens everything	every wavenumber decays
both, equal speeds	the flattening wins	every wavenumber decays
both, different speeds	a band opens	a length is born

Observation 6.2

A word on Paper 175. That paper split symmetry breaking three ways and wrote that spontaneous breaking does not occur in a finite system. That was about thermodynamic phase transitions. The breaking here is a linear instability of a deterministic equation, and it does occur in a finite domain — one phrase covering two matters again.

Observation 6.3

And the corpus gains another fence — linearity in Paper 171, integrality in 173, compactness in 177, the number of options in 184, the target space in 185, the tail of the error in 186, a finite exponential moment in 190, and here the ratio of two diffusion rates.

7. Conclusion

1. Reaction alone leaves the uniform state stable (Ver. 2.1): growth rate -0.2 .
2. Diffusion alone lets everything decay (Ver. 2.2).
3. Together, with a difference, a band opens (Ver. 3.1): **0.529** at a ratio of **30**.
4. The fence is a threshold in the ratio (Ver. 4.2): **3.41346** whatever d_1 is.
5. The length comes from $k^4 = \det J / (d_1 d_2)$ (Ver. 5.2): agreeing to 10^{-5} .
6. The wavelength grows with the ratio (Ver. 5.4): from **13.62** to **22.10**.

Closing: When a pattern is said to have a definite size, the next question is where that length came from. Not from the container and not from the initial condition — from a diffusion constant divided by a reaction rate, a quantity that was inside the equations all along. And what made the pattern was the very thing that flattens everything.

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