

“A Small World” Is Three Separate Claims

— Short Paths Come Free, and a Heavy Degree Tail Does Not Follow From Them — [Paper 192]

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ABSTRACT

Saying that a network is a small world bundles together at least three separate claims. This paper separates them and shows in numbers which does not follow from which. **NO NEW MATHEMATICAL THEOREM IS CLAIMED.** First, short paths come free. On 1000 nodes of average degree 10, a network with no structure whatever has a mean path length of **3.26** — so a short path is no evidence of design (Section 2). Second, and this is the core. Short paths and local clustering are carried by different edges. Rewiring 1% of a ring's edges drops the mean path from 50.45 to 8.75, that is to 17.3%, while the clustering falls only from 0.6667 to 0.6446, that is to 96.7% (Section 3). Third, a heavy degree tail does not follow from either. The 10%-rewired ring has a largest degree only 1.40 times the mean, while a network grown by preferential attachment reaches 14.50 times, with a largest degree of 142 (Section 4). Fourth, the fence is the second moment. With every average degree at about 10, the ratio of the second moment to the first came out as 10.00, 11.00, 10.10 and **22.20** (Section 5). Fifth, nothing is claimed here about any real network — only the four generated here were measured.

Scope of this paper: **NO NEW MATHEMATICAL THEOREM IS CLAIMED** — the Watts–Strogatz construction, preferential attachment, and the appearance of the second moment in a spreading threshold are all standard. Nothing is claimed about real networks, social, biological or technical — what the degree distribution of any real system is is not a question this paper touches. What is treated is measurements of generated networks. On a word: Paper 176 wrote that “small” becomes two different things under measure and under category. The “small” here is neither of those but a third use — one word splitting again, somewhere else. What is added is separating three claims by what carries each, measuring that a 1% rewiring moves the two independently, exhibiting a counterexample to the heavy tail following from short paths, and placing the fence at the second moment.

1. Notation and position

Four undirected networks on **1000** nodes of average degree **10**: a ring joining each node to its ten nearest on a circle, a purely random network, a ring with a fraction of edges rewired, and one grown by preferential attachment. The **mean path length** is measured by breadth-first search from **200** randomly chosen nodes, and the **clustering** is the average over nodes of the fraction of their neighbour pairs that are joined. All of it was computed for this paper.

Position: Paper 186 placed a fence at the tail of an error and Paper 190 placed the fence on rarity at a finite exponential moment. This paper adds how readily a network spreads to that row — and again the average is hiding it.

2. Short paths come free

Verification 2.1

network	mean path length	clustering
ring	50.4505	0.6667
purely random	3.2648	0.0100

(QED)

Observation 2.2

A network with no structure at all already has short paths. So an observation of the six-degrees kind is by itself no evidence of anything — it is what randomness supplies for nothing.

Observation 2.3

What is surprising is not the short path but that a short path and heavy clustering coexist. The ring is clustered and far; the random network is near and unclustered. Neither is a small world.

3. The core: different edges carry the two

Verification 3.1: Rewiring a fraction of the ring's edges.

fraction	mean path	clustering	path vs ring	clustering vs ring
0	50.4505	0.6667	1.0000	1.0000
0.001	34.0233	0.6654	0.6744	0.9980
0.01	8.7452	0.6446	0.1733	0.9668
0.05	5.3227	0.5823	0.1055	0.8734
0.2	3.8093	0.3356	0.0755	0.5034
1.0	3.2601	0.0080	0.0646	0.0119

(QED)

Observation 3.2

A one-percent rewiring cuts the path to 17% while 97% of the clustering survives. The two quantities are carried by different edges — a few shortcuts decide the distance, and many neighbourly ties decide the clustering.

Observation 3.3

So what the phrase “small world” names is neither property but the pair. Measuring one of them does not license the phrase.

4. A heavy degree tail does not follow

Verification 4.1

network	mean degree	largest	standard deviation	largest / mean
ring	10.00	10	0.00	1.00
random	10.00	21	3.17	2.10
rewired 10%	10.00	14	0.98	1.40
preferential	9.79	142	10.50	14.50

(QED)

Observation 4.2

The 10%-rewired ring is a perfectly good small world and its degrees are all much alike. The grown network has a node fourteen and a half times the average. A heavy tail does not follow from short paths — they are separate claims about separate things.

Observation 4.3

And what carries the heavy tail is how the network was grown — only a rule under which the already-well-joined get joined again produces it.

5. The fence is the second moment

Observation 5.1

For something spreading on a network, what governs the threshold is not the mean degree but the ratio of the second moment to the first.

Verification 5.2

network	mean degree	second moment over first	ratio to the mean
ring	10.000	10.000	1.000
random	10.000	11.004	1.100
rewired 10%	10.000	10.099	1.010
preferential	9.786	22.197	2.268

(QED)

Observation 5.3

Every average degree sits at about 10, and the quantity that enters the threshold differs by more than a factor of two. The average is hiding it.

Observation 5.4

This is the third time the corpus meets the same shape — Paper 186 put the fence at the tail of an error and Paper 190 put the fence on rarity at a finite exponential moment. In each, the average does not settle it and the tail does. Here too, equal average degrees do not give equal behaviour.

6. The census

Observation 6.1

what is called small	what carries it	caution
short paths	a few shortcuts	randomness gives it free
local clustering	many neighbourly edges	different edges from the paths
a heavy degree tail	how it was grown	does not follow from short paths
spreads readily	the second moment	not settled by the mean degree

Observation 6.2

These four are not a staircase. The first two are statements about distance, the third about a distribution, and the fourth about what happens on top. One phrase over all of them makes it seem that seeing one settles the rest.

Observation 6.3

A word on the word. Paper 176 wrote that “small” comes apart into measure zero and first category. The “small” here is neither — one word splitting again, somewhere else.

7. Conclusion

1. Short paths come free (Ver. 2.1): **3.26** in a purely random network.
2. Different edges carry the two (Ver. 3.1): a 1% rewiring leaves path at 17% and clustering at 97%.
3. A heavy tail does not follow (Ver. 4.1): largest over mean of 1.40 against 14.50.
4. The fence is the second moment (Ver. 5.2): **10.10** against **22.20** at the same average degree.
5. The four are not a staircase (Obs. 6.2).

Closing: When a network is called a small world, the next question is in which sense. Are the paths short, is it clustered, are there large nodes, or does something spread readily on it? The first is given free by randomness and the last is not settled by the average degree — the four have to be checked separately.

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