

# Two Coupled Costs Cannot Be Added

— The Ratio of Two Lengths Decides a Sign, and the Additive Estimate Misses by Exactly  $8/3$  — [Paper 193]

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— *From thinking to syncing.* —

## ABSTRACT

One free energy builds two lengths of its own, and only their ratio decides how it behaves. This paper computes those lengths exactly and then measures where an additive estimate fails. **NO NEW MATHEMATICAL THEOREM AND NO NEW PHYSICAL LAW IS CLAIMED.** First, both lengths are built from the coefficients. The length over which the order recovers and the length to which the field penetrates are both combinations of the free energy's own coefficients, and the size of the sample never appears (Section 2). Second, the cost of a wall can be had exactly. Quadrature gives 1.88561808316413 and the closed form  $4\sqrt{2}/3$  agrees with zero difference (Section 3). Third, and this is the core. The additive estimate — the wall costs, the expelled field saves — puts the crossing at 1.885618, while the exact answer is 0.707107 — a ratio of exactly  $8/3$ , agreeing to  $4 \times 10^{-31}$  (Section 4). Fourth, that gap is not a small correction. Between ratios of 0.707 and 1.886 the two views give opposite answers — the estimate is wrong precisely in the band where it would be used (Section 4). Fifth, the exact value  $1/\sqrt{2}$  is quoted. Obtaining it requires solving the two equations together, which this paper does not do (Section 4).

Scope of this paper: **NO NEW MATHEMATICAL THEOREM AND NO NEW PHYSICAL LAW IS CLAIMED** — the Ginzburg–Landau free energy, its two characteristic lengths, and the sign change of the surface energy at a ratio of  $1/\sqrt{2}$  are all standard. Nothing is claimed about any real material — which substance falls on which side is not a question this paper touches. And the exact crossing is not derived here — what was computed exactly is the cost of the wall, and  $1/\sqrt{2}$  is quoted. On Paper 191: that paper wrote that two ingredients doing nothing alone do the opposite together. This paper writes the estimating side of the same shape — cost them separately, add, and the crossing moves by  $8/3$ . What is added is setting out that both lengths are built from the coefficients, obtaining the wall's cost by quadrature and in closed form and checking that they agree, showing that the additive estimate's error is exactly  $8/3$ , and writing down the band in which the two answers are opposite.

## 1. Notation and position

Take a free energy for an order parameter  $f$  with a quadratic coefficient  $-a$ , a quartic coefficient  $b$ , a gradient stiffness  $g$ , and a coupling  $q$  to a field, with  $f$  normalised to **0** on the disordered side and **1** on the ordered side. All numbers were computed for this paper, with the quadrature done in extended precision.

**Position:** Paper 191 treated a length appearing where there was none. This paper treats two lengths appearing, so that only their ratio matters — and measures what a naive estimate of that ratio does.

## 2. Both lengths are built from the coefficients

**Verification 2.1:** Vary the coefficients and watch  $\xi = \sqrt{g/a}$ , the length over which the order recovers, and  $\lambda = \sqrt{b/(q^2a)}$ , the length to which the field penetrates.

| $a$ | $b$ | $g$ | $q$ | $\xi$ | $\lambda$ | ratio      |
|-----|-----|-----|-----|-------|-----------|------------|
| 1   | 1   | 1   | 1   | 1.0   | 1.0       | 1.0        |
| 1   | 1   | 4   | 1   | 2.0   | 1.0       | <b>0.5</b> |
| 1   | 4   | 1   | 1   | 1.0   | 2.0       | <b>2.0</b> |
| 4   | 1   | 1   | 1   | 0.5   | 0.5       | 1.0        |
| 1   | 1   | 1   | 2   | 1.0   | 0.5       | 0.5        |

(QED)

### Observation 2.2

Both lengths are combinations of the coefficients. The size of the sample appears nowhere — just as the container did not appear in Paper 191. And as the fourth row shows, shrinking both equally leaves the ratio unmoved.

### 3. The cost of a wall, exactly

**Observation 3.1**

The order parameter recovers as  $f(x) = \tanh\left(x/(\sqrt{2}\xi)\right)$ , and the cost per unit area is the integral of the gradient term plus the potential term.

**Verification 3.2**

| $\xi$ | by quadrature     | $4\sqrt{2}\xi/3$  | difference |
|-------|-------------------|-------------------|------------|
| 1     | 1.88561808316413  | 1.88561808316413  | 0          |
| 2     | 3.77123616632825  | 3.77123616632825  | 0          |
| 0.5   | 0.942809041582063 | 0.942809041582063 | 0          |

(QED)

**Observation 3.3**

A first draft of this paper wrote the closed form as  $2\sqrt{2}/3$ , and it disagreed with the quadrature by a factor of two. The disagreement is what caught the mistake. Obtain both the closed form and the integral and set them against each other — with only one of them, the error passes quietly.

### 4. The core: cost them separately, add, and miss by 8/3

**Observation 4.1**

The tempting estimate runs: **an interface pays the cost of the wall and saves, in exchange, the energy of the field it keeps out over a depth  $\lambda$ . So the surface energy should vanish where those balance.**

**Verification 4.2**

| quantity                            | value                 |
|-------------------------------------|-----------------------|
| crossing from the additive estimate | 1.885618083           |
| the exact crossing                  | <b>0.707106781</b>    |
| ratio between them                  | <b>2.666666667</b>    |
| <b>8/3</b>                          | 2.666666667           |
| difference between the two          | $3.9 \times 10^{-31}$ |

(QED)

**Observation 4.3**

The additive estimate misses by exactly  $8/3$ . And this is not a small correction.

**Verification 4.4:** What each view answers.

| ratio $\lambda/\xi$ | the additive estimate | the exact answer  |
|---------------------|-----------------------|-------------------|
| 0.5                 | keeps it out          | keeps it out      |
| 0.707107            | keeps it out          | keeps it out      |
| 1.0                 | keeps it out          | <b>lets it in</b> |
| 1.3                 | keeps it out          | <b>lets it in</b> |
| 1.886               | lets it in            | lets it in        |
| 3.0                 | lets it in            | lets it in        |

(QED)

#### **Observation 4.5**

Between 0.707 and 1.886 the two answers are opposite. And that band is exactly where the question is actually asked — the estimate is wrong precisely where one would want to use it.

#### **Observation 4.6**

The exact value  $1/\sqrt{2}$  is quoted. What this paper computed exactly is the cost of the wall; obtaining the crossing requires solving the equations for the order parameter and the field together, which is not done here.

### **5. Why they cannot be added**

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#### **Observation 5.1**

The reason is not about the material but about the coupling. The order parameter keeps the field out and the field pushes the order parameter down. So the wall's cost must be computed with the field present, and the field's saving must be computed with the order still recovering. Adding two numbers each obtained alone means each was computed in a world where the other was absent.

#### **Observation 5.2**

Paper 191 has the same shape. There, neither reaction nor diffusion makes a pattern alone, and together they do the opposite of either. Here, costing them alone and adding moves the crossing by  $8/3$ . The two are the same caution from opposite sides — in a coupled system the behaviour of the whole does not come from the behaviour of the parts.

**Observation 5.3**

And when the field is let in, the reason it enters in quanta is a different one. The winding of the phase must be a whole number, which is Paper 185's classification exactly — the target space is a circle, so the winding is an integer. That the ratio decides a sign is about energy; that the entry is quantised is about phase. Different roots.

**6. The census: lengths that do not come from outside**

**Observation 6.1**

| where      | what length              | built from                   | what never appears         |
|------------|--------------------------|------------------------------|----------------------------|
| Paper 191  | the width of a stripe    | diffusion over rate          | the container              |
| this paper | recovery and penetration | combinations of coefficients | the sample                 |
| Paper 182  | none at all              | —                            | only the dimension matters |

**Observation 6.2**

In both papers the length was not given but built. And here, because two lengths are built, only the dimensionless ratio does any work.

**7. Conclusion**

1. Both lengths are built from the coefficients (Ver. 2.1): the sample never appears.
2. The wall's cost is exact (Ver. 3.2): quadrature and  $4\sqrt{2}\xi/3$  differ by zero.
3. A first draft's closed form was off by two (Obs. 3.3): the comparison caught it.
4. The additive estimate misses by  $8/3$  (Ver. 4.2): 1.885618 against 0.707107.
5. In that band the answers are opposite (Ver. 4.4).
6. The exact crossing is quoted (Obs. 4.6).

**Closing: When told that a behaviour switches where one cost balances another saving, the next question is whether the two are independent. If they are not, the numbers obtained separately cannot be added — because each was computed in a world where the other was absent. Here the price of doing so came out as exactly  $8/3$ .**

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