

“Pressure” Is Four Separate Quantities, and Sharing a Unit Guarantees Nothing

— The Separator Is What Must Be Moved: One of Them Ignores Temperature, and One Has No Equation of State — [Paper 196]

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August 2026

— *From thinking to syncing.* —

ABSTRACT

The thermodynamic pressure, the kinetic pressure, the degeneracy pressure, the radiation pressure and the pressure of an incompressible flow all carry the same unit. This paper shows numerically that sharing a unit guarantees nothing. NO NEW MATHEMATICAL THEOREM AND NO NEW PHYSICAL LAW IS CLAIMED. First, the degeneracy pressure ignores the temperature. Cooling a free Fermi gas to $T/E_F = 10^{-3}$, the classical nT falls to 0.0076 while the pressure stays at 3.0385, and the ratio reaches 400.0 and keeps growing (Section 2). Second, there is no force in it at all. The Hamiltonian used is kinetic energy and nothing else: no interaction between particles and no field — and still the pressure does not vanish (Section 2). Third, one energy density gives several different pressures. An isotropic photon gas gives $u/3$, an absorbed beam gives u , a reflected beam gives $2u$, and a beam at 60 degrees gives $u/4$ — a range of eight from one u (Section 3). Fourth, this is the core. The pressure of an incompressible flow has no equation of state. It solves an elliptic equation, it is fixed only up to a constant, and it is decided by the whole field — changing the flow in one cell of a 256×256 grid changed the pressure in 65536 of 65536 cells, that is 100.00% (Section 4). Fifth, doubling the density gives different answers. The degeneracy pressure grows by 3.174794 and the incompressible pressure by exactly two (Section 5).

Scope of this paper (scope note): NO NEW MATHEMATICAL THEOREM AND NO NEW PHYSICAL LAW IS CLAIMED — Fermi–Dirac statistics, radiation pressure, and the pressure Poisson equation of incompressible flow are all standard. No claim is made about which pressure is the “real” one — all five enter the momentum balance on equal terms. Nothing is claimed about any real star or fluid — every number here comes from a dimensionless model. The relativistic degenerate regime is not treated — the calculation here is non-relativistic. Relation to Paper 141: How the exponent of the degeneracy pressure produces an upper bound is the subject of Paper 141 — this paper does not use that exponent, only the fact that the answers differ. Relation to Paper 145: 145 treated pressure as a source of gravity — that was about mechanism, and this is a census. What is added is measuring the ratio between the degenerate and classical pressures diverging across four decades, setting out the eightfold range obtained from one energy density, counting that a one-cell change reaches 100% of the grid, and placing the two answers to doubling the density side by side.

1. Notation and Position

Pressure is what appears in a momentum balance as force per unit area. Its unit is $\mathbf{Pa} = \mathbf{N/m^2} = \mathbf{J/m^3} = \mathbf{kg/(ms^2)}$, and all five quantities below carry it. For the Fermi gas, $\hbar = m = 1$; the quadrature and the bisection are both written here.

Position: Paper 157 took a census of the 2s in this body of work, and Paper 194 took a census of the operation of averaging. This paper takes a census of “pressure” — and the question is not which one is correct, but what must be moved for each of them to move.

2. The Degeneracy Pressure Ignores Temperature, and Has No Force in It

Check 2.1: hold a free Fermi gas at number density $n = 1$ and lower the temperature, measuring the pressure. At each temperature the chemical potential was found by bisection and the pressure by quadrature. Here $k_F = 3.897777$, $E_F = 7.596333$, and the closed form at $T = 0$ is $P = \frac{2}{5}nE_F = 3.038533248$.

T/E_F	Fermi P	Classical nT	Ratio	$P/P(0)$
1	8.592664900	7.596333121	1.1312	2.827898923
0.3	3.986742518	2.278899936	1.7494	1.312061509
0.1	3.161508291	0.759633312	4.1619	1.040471844
0.03	3.049764111	0.227889994	13.3826	1.003696146
0.01	3.039782610	0.075963331	40.0164	1.000411173
0.003	3.038645706	0.022788999	133.3383	1.000037011
0.001	3.038545744	0.007596333	400.0016	1.000004112

Observation 2.2

As the temperature falls the classical pressure disappears and this one does not. The ratio passes 400 and is still growing, and $P/P(0)$ has settled at 1.000004. The explanation that “there is pressure because the molecules are moving about hotly” does no work at all here.

Observation 2.3

And there is no force in it at all. The Hamiltonian used above is $k^2/2$ per particle and nothing else: no potential between particles, no external field, no interaction term. And still $P = 3.038533248$ survives as $T \rightarrow 0$. What produces this pressure is not a force but a rule forbidding two particles in one state.

3. One Energy Density Gives Several Different Pressures

Check 3.1: hold the energy density u fixed and vary how the light arrives and what the wall does. For the isotropic case the average of $\cos^2$ over the sphere was taken numerically.

Situation	P/u
Isotropic photon gas	0.333335 (against $1/3 = 0.333333$)
Beam at 60 degrees, fully absorbed	0.250000
Beam at normal incidence, fully absorbed	1.000000
Beam at normal incidence, fully reflected	2.000000

Observation 3.2

From one u , the answer runs from 0.25 to 2.00: a factor of eight. What is being moved here is not the amount of light but the angular distribution and what the wall does. So the phrase “radiation pressure” by itself does not name a number either.

Observation 3.3

The $1/3$ is not peculiar to photons. It is the average of $\cos^2$ when an isotropic momentum flux in three dimensions is projected onto a surface, and it is the same whatever the particle is — like the 2s censused in Paper 157, this 3 comes from the dimension.

4. The Pressure of an Incompressible Flow Has No Equation of State

In an incompressible flow the density is declared not to move, so the pressure cannot be obtained from the density or the temperature. Instead it appears as **the multiplier that enforces $\nabla \cdot \mathbf{u} = 0$** , and it is fixed by the Poisson equation obtained by taking the divergence.

Check 4.1: place the Taylor–Green vortex $\mathbf{u} = (\sin x \cos y, -\cos x \sin y)$ on a 256×256 periodic grid. By finite differences the largest $|\nabla \cdot \mathbf{u}|$ is 1.876×10^{-14} . Substituting the candidate $p = \frac{1}{4}(\cos 2x + \cos 2y)$ into the pressure Poisson equation, the largest discrepancy between the two sides is 1.488×10^{-12} .

Observation 4.2

No time derivative appears in that equation. It is elliptic, neither parabolic nor hyperbolic — so the pressure does not propagate: it is settled at that instant.

Check 4.3 (fixed only up to a constant): adding a constant to p leaves the flow unchanged. Compare the gradients.

Constant added	Largest $ \nabla(p + c) - \nabla p $
0	0
1	4.552×10^{-15}
10^6	2.178×10^{-9}

The residues are floating-point rounding: adding 10^6 leaves the flow the same. Do that to a thermodynamic pressure and the state changes — here nothing happens at all.

Check 4.4 (this is the core): change the flow in **one cell** of the grid, solve the pressure Poisson equation again, and count how far the pressure changed.

Quantity measured	Value
Grid	$256 \times 256 = 65536$ cells
Cells changed in the flow	1
Cells whose pressure moved by more than 10^{-9}	65536 (100.00%)
Change at the neighbouring cell	2.049×10^{-2}
Change at the farthest cell	1.628×10^{-3} (7.9% of the neighbour)
Smallest change anywhere on the grid	3.240×10^{-8}

(The box is periodic, so distance stops growing past 128 cells.)

Observation 4.5

A change in one cell reached 100% of the grid. This pressure is not a local quantity — the pressure at a point is fixed not by the state at that point but by the entire flow. A real fluid, of course, is not incompressible, and information travels at the speed of sound. Incompressibility is the approximation that sends the sound speed to infinity, and this instantaneity is its price — having lost its equation of state, the pressure took on the job of maintaining a constraint instead.

5. Doubling the Density, the Answers Do Not Agree

Check 5.1: in a free Fermi gas at very low temperature, raise the number density from **1** to **2**.

System	Factor by which the pressure grows	Value
Free Fermi gas at low temperature	$9.646757493/3.038545744$	3.174794
Incompressible flow	density enters only as a multiplier	2.000000

Observation 5.2

The intuition that twice the density means twice the pressure is right in only one of these. Why the exponent behind that factor takes the value it does, and how it produces an upper bound on the mass of a star, is the subject of Paper 141 — this paper does not use that exponent, only the fact that the two answers differ.

6. The Separator: What Must Be Moved

Observation 6.1

line the five up by what moves them and what they ignore.

Name	What moves it	What it ignores
Thermodynamic	the volume, at fixed temperature	—
Kinetic	the velocity distribution	the interactions
Degeneracy	the number density (and $\hbar$)	the temperature
Radiation	the angular distribution and the wall	the medium
Incompressible flow	the whole velocity field, at once	the equation of state

Observation 6.2

The five carry the same unit, enter the momentum balance on the same terms, and not one of them is the same function. The first two agree for an ideal gas, the third survives the loss of temperature, the fourth produces an eightfold range from one energy density, and the fifth is not a function of the state at all.

Observation 6.3

So “same unit, therefore same quantity” does not hold. What the unit guarantees is only that they may be added — and indeed inside a star the gas pressure, the radiation pressure and the degeneracy pressure are added. Being addable and being the same mechanism are different things.

7. The Census and the Close

What this paper checked:

1. The degeneracy pressure ignores temperature (Check 2.1): at $T/E_F = 10^{-3}$ the ratio is 400.0 and $P/P(0)$ is 1.000004.
2. There is no force in it at all (Observation 2.3): $P = 3.038533248$ from a Hamiltonian of kinetic energy alone.
3. An eightfold range from one energy density (Check 3.1): from 0.25 to 2.00.
4. The incompressible pressure has no equation of state (Checks 4.1, 4.3, 4.4): elliptic, fixed up to a constant, and a one-cell change reaching 100.00%.
5. Doubling the density gives different answers (Check 5.1): 3.174794 against 2.000000.
6. The separator is what must be moved (Observation 6.1).

Close: The single word “pressure” puts one label on five different functions. The label is shared because they all stand in the same place in a momentum balance, not because they stand there for the same reason. One is there because molecules move, one because two particles cannot occupy one state, one because light bounces off, and the last because without it a constraint would break. That the units agree says nothing whatever about those four reasons — what a unit guarantees is addition, and being addable is not being the same.

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