

The Counts of Critical Points Move a Great Deal, and Their Alternating Sum Never Moves at All

— On a Torus the Total Swings from 290 to 366 While the Alternating Sum Stays at 0: the Floor Is 4, and
Chance Never Got Below 282 — [Paper 203]

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August 2026

— *From thinking to syncing.* —

ABSTRACT

Put a function on a surface and it fixes how many minima, saddles and maxima there are. This paper shows, by integer counting with no tolerance anywhere, that all three move a great deal while the alternating sum never moves at all. **NO NEW MATHEMATICAL THEOREM AND NO NEW LAW IS CLAIMED.** First, the parts move. Running 100 random functions on a 24 by 24 triangulated torus, c_0 spreads over 72 to 94, c_1 over 145 to 183, c_2 over 72 to 91, and the total over 290 to 366 (Section 2). Second, the alternating sum does not move. Across all 100, the set of values taken by $c_0 - c_1 + c_2$ has exactly one element, 0 (Section 2). Third, the same procedure returns a different fixed number on a sphere. Across three levels of refinement the total ranges from 16 to 396, and the alternating sum is always 2 (Section 3). Fourth, this is the core. The floor exists and chance never reaches it. The floor on the torus is 4 — four thousand random functions never got below **282**, while one built on purpose reached 4 at the first attempt (Section 4). Fifth, every excess cancels. Setting the excess over the floor side by side, $e_0 - e_1 + e_2 = 0$ in every trial — one extra minimum or maximum is always paid for with one extra saddle (Section 5).

Scope of this paper (scope note): NO NEW MATHEMATICAL THEOREM AND NO NEW LAW IS CLAIMED — the Morse inequalities, Banchoff's discrete index and the Euler characteristic are all standard. The Morse inequalities are not proved — this paper only counts what they permit and what they forbid. Random values are noise as functions — values drawn independently at the vertices of a grid have no correlation between neighbours, so a large number of critical points is only to be expected, and what is surprising is not the number but that the alternating sum does not move even in that noise. Only two surfaces are treated — a torus and a sphere; no general statement about surfaces is made. Relation to Paper 128: 128 treated Poincaré–Hopf and wrote that the sum of the indices is the Euler characteristic — this paper counts not the sum but how free the individual terms are. Relation to Paper 151: 151 verified a procedure for computing Betti numbers from the ranks of boundary matrices — this paper only uses those numbers as a floor and does not enter the computation. What is added is setting the spread of the parts and the fixity of the sum inside the same trials, confirming across three refinements that the same procedure returns a different fixed number on a different surface, measuring over four thousand trials that chance does not reach the floor, and checking trial by trial that the excess cancels.

1. Notation and Position

Assign distinct values to the vertices of a triangulated surface. Among the **link** of a vertex v , the ring of vertices around it, call those with smaller values **lower**. If there is no lower vertex, v is a **minimum**; if all are lower, a **maximum**; and if the lower ones fall into k separate runs around the ring, count $k - 1$ **saddles** there. This counting uses integers only and **carries no tolerance whatever**. Write c_0 , c_1 , c_2 for the totals of minima, saddles and maxima.

Position: Paper 128 treated Poincaré–Hopf and wrote that the sum of the indices of a vector field is the Euler characteristic. What this paper counts is not the sum but how free the individual terms making it up are.

2. The Torus: the Parts Move, the Alternating Sum Does Not

Check 2.1: on a **24** by **24** triangulated torus with **576** vertices, place **100** random assignments of values.

Trial	c_0	c_1	c_2	$c_0 - c_1 + c_2$
1	80	161	81	0
2	85	171	86	0
3	81	168	87	0
4	87	172	85	0
5	88	163	75	0
6	76	155	79	0

Check 2.2: gather all 100.

Quantity measured	Value
Range of c_0	72 to 94
Range of c_1	145 to 183
Range of c_2	72 to 91
Range of the total	290 to 366
Values taken by $c_0 - c_1 + c_2$	0 only

Observation 2.3

Each of the three swings by more than 20, and the total by 76. The one number made by adding and subtracting them was the same all 100 times. Not “close to the same” — the counting is in integers, so it is exactly the same.

3. The Sphere: the Same Procedure Returns a Different Fixed Number

Check 3.1: on a subdivided icosahedron, repeat at three levels of refinement, **40** trials each.

Subdivision	Vertices	$c_0 - c_1 + c_2$	Range of the total
1	42	2 only	16 to 30
2	162	2 only	74 to 104
3	642	2 only	342 to 396

Observation 3.2

Multiplying the vertices by fifteen and the total by twenty leaves the alternating sum at 2. It is 0 for the torus and 2 for the sphere — the procedure is the same and only the number returned depends on the surface.

Observation 3.3

This paper does not derive what the 0 and the 2 are — that they are the Euler characteristic is a standard fact, and what is checked here is the behaviour of not moving.

4. The Core: the Floor Exists and Chance Never Reaches It

The Morse inequalities say that c_i cannot fall below the Betti number b_i . On the torus $b_0, b_1, b_2 = 1, 2, 1$, so at least 4 critical points are required.

Check 4.1:

How the function was built	(c_0, c_1, c_2)	Total
Random values, the least of 4000 trials	(73, 141, 68)	282
One cosine in each direction	(1, 2, 1)	4
The floor (the Betti numbers)	(1, 2, 1)	4

Observation 4.2

The floor is 4, and a function built on purpose lands on it at the first attempt. Random functions, over four thousand trials, came down only to 282 — seventy times the floor.

Observation 4.3

This must not be read as “randomness makes a poor function”. Independent values at the points of a grid are noise, and with no correlation between neighbours it is only to be expected that roughly half the vertices are critical. What is surprising is not the number — it is that even inside that noise the alternating sum never moved once. Two hundred and eighty-two meaningless comings and goings cancelled exactly.

5. Every Excess Cancels

Check 5.1: on a 16 by 16 torus, set out the excess over the floor, $e_i = c_i - b_i$.

e_0	e_1	e_2	Total excess	$e_0 - e_1 + e_2$
30	67	37	134	0
37	74	37	148	0
32	68	36	136	0
33	71	38	142	0
33	67	34	134	0
37	77	40	154	0

Observation 5.2

One extra minimum or maximum is paid for with exactly one extra saddle. An extra critical point is never born alone — it always brings a partner that cancels it. There is freedom to leave the floor, and the leaving has a direction.

6. The Separator and the Fence

Observation 6.1 (the separator)

What separates what moves from what does not is whether it belongs to the function or to the surface. The three counts are fixed by the function that was placed — change the function and they move by tens. The alternating sum is fixed by the surface — whatever function is placed, the same number comes back. Two kinds of quantity are mixed together inside the same trial.

Observation 6.2 (the fence)

The fence around “count the critical points and you learn the shape” is taking the alternating sum. Inside the fence the answer is settled by the surface alone; outside it — that is, looking at c_0 alone, or c_1 alone — the answer is swallowed by the habits of the function. The swing from 290 to 366 is the size of that swallowing.

Observation 6.3

The floor, by contrast, lives outside the fence — $c_i \geq b_i$ is a statement about each c_i separately, and it bites before any sum is taken. So the two constraints are different things: one puts a floor under each term, and the other fixes a combination.

7. The Census and the Close

What this paper checked:

1. The parts move a great deal (Check 2.2): the total runs from 290 to 366.
2. The alternating sum never moves (Check 2.2): 0 in all 100, exactly.
3. A different surface, a different fixed number (Check 3.1): always 2 on the sphere, even at fifteen times the vertices.
4. The floor is 4 and the least by chance is 282 (Check 4.1): first attempt on purpose, seventy times the floor by chance.
5. What is surprising is not the number but that the sum holds inside noise (Observation 4.3).

6. Every excess cancels (Check 5.1): $e_0 - e_1 + e_2 = 0$ in every trial.

7. The fence is taking the alternating sum (Observation 6.2).

Close: Place one function on a surface and more than three hundred critical points can appear. Almost all of them are nothing but the habits of the function placed — place another and the counts move by tens. And yet, the moment one fixed way of adding and subtracting them is chosen, all of that disappears and a number belonging to the surface remains. What this paper measured is the exactness of that disappearance — two hundred and eighty-two meaningless comings and goings cancelled without leaving a remainder of one. When something is counted, the way to separate what belongs to the object from what belongs to the instrument is to change the instrument and count again. Only what did not move belonged to the object.

References

1. Morse, M. (1934). *The Calculus of Variations in the Large*. Providence: American Mathematical Society.
2. Banchoff, T. (1967). *Critical Points and Curvature for Embedded Polyhedra*. Journal of Differential Geometry, 1(3).
3. Milnor, J. (1963). *Morse Theory*. Princeton: Princeton University Press.
4. Poincaré, H. (1885). *Sur les courbes définies par les équations différentielles*. Journal de Mathématiques Pures et Appliquées, 1.
5. Hopf, H. (1927). *Vektorfelder in n-dimensionalen Mannigfaltigkeiten*. Mathematische Annalen, 96(1).
6. Yamagishi, Y. (2026). $4\pi = 2\pi\chi$ Holds in Three Dimensions and Nowhere Else [Paper 128]. Zenodo. 10.5281/zenodo.22079576
7. Yamagishi, Y. (2026). *A Two-Set Cover Returns the Wrong Answer for the Circle* [Paper 151]. Zenodo. 10.5281/zenodo.22084096
8. Yamagishi, Y. (2026). *A Fixed Point Is Not One Theorem* [Paper 161]. Zenodo. 10.5281/zenodo.22092530
9. Yamagishi, Y. (2026). “Straight” Does Not Mean “Shortest”, and There Are Two Fences [Paper 198]. Zenodo. 10.5281/zenodo.22107954
10. Yamagishi, Y. (2026). *The Two Hundredth Census* [Paper 200]. Zenodo. 10.5281/zenodo.22108829
11. Yamagishi, Y. (2026). *A Connection Is Not Derived from the Metric* [Paper 202]. Zenodo. 10.5281/zenodo.22109349

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