

The Spaces Converge and the Dimension Does Not

— A Thin Cylinder Comes Within ϵ of a Segment While Its Dimension Stays **2** However Small ϵ Is — [Paper
206]

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— *From thinking to syncing.* —

ABSTRACT

When a sequence of spaces approaches one space, the dimension does not approach anything. This paper measures that on a sequence of thin cylinders approaching a segment, and shows by integer counting that dimension is not continuous in the limit. **NO NEW MATHEMATICAL THEOREM AND NO NEW LAW IS CLAIMED.** First, the spaces really do converge. A cylinder of radius ϵ stands ϵ from the segment at its core, and its area $2\pi\epsilon$ goes to zero with it — while its length never moves from 1 (Section 2). Second, the dimension stays at **2** regardless. Fitted below the circumference, the exponent runs 1.9650, 1.9799, 1.9843, 1.9852, 1.9814 as ϵ falls from 0.2 to 0.0005 (Section 3). Third, this is the core. One and the same set returns a different dimension at a different scale. Asked of a cylinder of radius 0.01, the exponent is 1.0965 at scale 0.3, 0.8505 at 0.1, 2.0435 at 0.02 and 1.9371 at 0.002 — the question “what is the dimension of this set” has no answer until the scale is named (Section 4). Fourth, how far it drops is set by how many directions were squeezed. A cylinder and a torus $(1, \epsilon)$ drop by one, while a torus (ϵ, ϵ) and a box $(1, \epsilon, \epsilon)$ drop by two — the measured exponents are 1.9853, 1.9858, 1.9717 and 2.9716 (Section 5). Fifth, the first way of counting failed. Sampling the surface with points and laying a grid on the surrounding space made the slope oscillate with period three from the grid offset, and at fine scales the sample ran out and returned 1.5169 where it should have returned **2** (Section 6).

Scope of this paper (scope note): NO NEW MATHEMATICAL THEOREM AND NO NEW LAW IS CLAIMED — Gromov–Hausdorff convergence, collapsing and covering dimension are all standard. The Gromov–Hausdorff distance is not computed — what is measured here is the Hausdorff distance inside one embedding, which is only an upper bound on the true distance. Curved spaces are not treated — every number comes from flat rectangles and their gluings, and collapsing under a curvature bound is not discussed. No census of the word “dimension” is taken — how many ways that word is used is the subject of Paper 94. Relation to Paper 204: 204 measured that the power in an eigenvalue count is the dimension and set the fence at looking from high enough up — this paper measures the other face of that fence: looking at a fine enough scale. Relation to Paper 197: 197 asked what survives a limit — here the length survives and neither the area nor the dimension does. What is added is counting the covering exactly, with no sampling, measuring at five thicknesses that the exponent separates into 1 and 2 above and below the circumference, showing at four scales that one set returns different answers, and confirming across five spaces that the drop is set by the number of squeezed directions.

1. Notation and Position

Take a cylinder of radius ϵ and length 1. It is **flat**: intrinsically a rectangle of $2\pi\epsilon$ by 1, glued along the first side. The **covering number** $N(\delta)$ at scale δ is counted **exactly** as $\lceil 2\pi\epsilon/\delta \rceil \times \lceil 1/\delta \rceil$, and the exponent is fitted by least squares of $\log N$ against $\log(1/\delta)$. **No sampling is used anywhere.**

Position: Paper 204 measured that the power in an eigenvalue count is the dimension, and set the fence at looking from high enough up. This paper looks at that fence from the side of length — and measures that without naming a scale there is no answer called the dimension.

2. The Spaces Really Do Converge

Check 2.1: compare the cylinder with the segment at its core.

ϵ	Distance to the segment	Area $2\pi\epsilon$	Length
0.3	0.300000	1.884956	1 (unchanged)
0.1	0.100000	0.628319	1 (unchanged)
0.03	0.030000	0.188496	1 (unchanged)
0.01	0.010000	0.062832	1 (unchanged)
0.003	0.003000	0.018850	1 (unchanged)

Observation 2.2

What is measured here is the Hausdorff distance inside one embedding — the Gromov–Hausdorff distance is at most this, and its true value is not computed here. All that is claimed is that it goes to zero.

3. And the Dimension Stays at 2

Check 3.1: write $L = 2\pi\epsilon$ for the circumference and fit the exponent over a decade well below L (from $L/100$ to $L/10$) and over a decade well above it (from $3L$ to $30L$).

ϵ	$L = 2\pi\epsilon$	Fitted above L	Fitted below L
0.2	1.256637	(L is too large)	1.9650
0.05	0.314159	(L is too large)	1.9799
0.01	0.062832	0.7728	1.9843
0.002	0.012566	0.9451	1.9852
0.0005	0.003142	0.9832	1.9814

Observation 3.2

The lower column sits on 2 however small ϵ is made. The upper column approaches 1 as ϵ shrinks — 0.7728, 0.9451, 0.9832. The thinner the cylinder, the wider the range over which it answers 1.

4. The Core: One Set Returns a Different Dimension at a Different Scale

Check 4.1: ask a single cylinder of radius 0.01, circumference 0.062832, at four scales.

Scale δ	Exponent there	Where it sits
0.3000	1.0965	far coarser than the circumference
0.1000	0.8505	coarser
0.0200	2.0435	near the circumference
0.0020	1.9371	finer

Check 4.2: follow the same cylinder as the scale is refined continuously.

δ	$N(\delta)$	Local exponent
0.180838	6	1.1961
0.091799	11	0.9394
0.065405	16	1.1053
0.046600	44	2.9841 (the circumference was crossed)
0.023655	129	2.1613
0.008556	936	1.8261
0.003094	6804	1.9905
0.001571	26117	2.0205

Observation 4.3

One set returns two answers. It is 1 at coarse scales and 2 at fine ones, and the switch happens at the cylinder's own circumference. So the question “what is the dimension of this set” has no answer until the scale is named.

Observation 4.4

This is not a matter of approximation. The counting is in integers and $N(\delta)$ is exact — both the 1 and the 2 are correct about the same set.

5. How Far It Drops Is Set by How Many Directions Were Squeezed

Check 5.1: with $\epsilon = 10^{-3}$, fit the exponent below the smallest side and compare with the limit space.

Space	Exponent at $\epsilon > 0$	Limit space	Drop
Cylinder of radius ϵ	1.9853	a segment	1
Flat torus $(1, \epsilon)$	1.9858	a circle	1
Flat torus (ϵ, ϵ)	1.9717	a point	2
Box $(1, \epsilon, \epsilon)$	2.9716	a segment	2
Box $(1, 1, \epsilon)$	2.9858	a square	1

Observation 5.2

Squeeze one direction and it falls by one; squeeze two and it falls by two. The drop is not a fixed number but how many directions were squeezed. And in the limit it can be taken all the way down to a point.

6.The First Way of Counting Failed

The first run of this paper scattered points over the cylinder and laid a grid on the surrounding three-dimensional space. That broke for two reasons.

What happened	The symptom
The offset of the grid	the local exponent oscillated with period three: 1.1023, 0.8550, 0.6986
The sample ran out at fine scales	it returned 1.5169 where it should have returned 2

Observation 6.1

The repair was to stop sampling and count exactly. The cylinder is flat, so intrinsically it is a rectangle and the covering number is exactly a product of ceilings. Differences between two neighbouring scales were dropped as well — two nearby scales can fall inside the same step of a ceiling, where the exponent reads zero or spikes. Fitting over a full decade removes that too.

Observation 6.2

Had the first numbers been written down, the report would have said “the dimension is 1.5”, a number belonging to nobody. It came from the sampling and from the grid, and yet it looks like a property of the set — that is what makes this kind of error dangerous.

7. The Separator and the Fence

Observation 7.1 (what survives)

Quantity	In the limit	Why
The distance	converges	that is what convergence means
The length	survives	1 from beginning to end
The area	does not survive	$2\pi\epsilon$ goes to zero
The dimension	does not converge	2 at every ϵ , 1 in the limit
The topology	does not survive	an annulus becomes an interval

Observation 7.2 (the separator)

What separates what survives from what does not is whether it uses the information in the squeezed direction. The length looks only at the direction that is not squeezed, so it survives. The area, the dimension and the topology all count the squeezed direction, and they vanish with it.

Observation 7.3 (the fence)

The fence around “dimension is a property of the space” is looking at a fine enough scale. Inside the fence the answer is 2 and outside it is 1 — and the fence sits in a different place for each space, at its own circumference for a cylinder. Paper 204 measured the same fence from the other side: there it was “looking from high enough up” and here it is “looking at a fine enough scale”, and they are one demand.

8. The Census and the Close

What this paper measured:

1. The spaces converge (Check 2.1): distance and area to zero, length fixed at 1.
2. The dimension stays at 2 (Check 3.1): 1.9650 through 1.9814.
3. One set, two answers by scale (Check 4.1): 1.0965 and 2.0435.
4. The switch is at its own circumference (Check 4.2): the local exponent spikes to 2.9841 there.
5. The drop counts the squeezed directions (Check 5.1): one or two, down to a point.
6. The first way of counting failed (Section 6): grid oscillation and an exhausted sample gave 1.5.
7. The fence is looking at a fine enough scale (Observation 7.3).

Close: A thin cylinder comes as close to a segment as one likes. The distance goes to zero, the area goes to zero, and to the eye it becomes indistinguishable from a segment. And still, at every stage it is two-dimensional — except that the two-dimensionality is visible only when looked at below its own circumference. Dimension turned out not to be a number belonging to a space alone, but a number belonging to a space together with a scale. And at the end of the limit no scale is fine enough — because what was approached has no circumference left.

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