

There Are Three Tools for Writing Polarization, and One of Them Cannot Write Unpolarized Light

— Two Hundred Thousand Jones Vectors All Return a Degree of Polarization of 1: the Same Two Beams Give 1 and 0 According to How They Are Added — [Paper 207]

Yuuki YAMAGISHI (山岸 勇輝)

Yamagishi Complexity Torus Lab, Japan

August 2026

— *From thinking to syncing.* —

ABSTRACT

There are three tools for writing down the polarization of light — the Jones vector, the Stokes parameters, and the Poincaré sphere. This paper shows numerically that the first cannot represent unpolarized light, and writes that what separates the three is what gets added. **NO NEW MATHEMATICAL THEOREM AND NO NEW PHYSICAL LAW IS CLAIMED.** First, a Jones vector returns a degree of polarization of 1 and nothing else. Making 200000 of them at random, the smallest is 0.9999999999999999 and the largest 1.0000000000000001 , the departure from 1 staying within 6.66×10^{-16} (Section 2). Second, this is the core. The same two beams give 1 and 0 according to how they are added. Adding equal horizontal and vertical beams as amplitudes gives light polarized at 45 degrees, of degree 1; adding them as intensities gives unpolarized light, of degree 0 — and only the first can be written with a Jones vector (Section 3). Third, mixtures fill the interior of the ball. Mixing with weights 1.00, 0.90, 0.75, 0.60, 0.50 gives degrees 1.000000, 0.800000, 0.500000, 0.200000, 0.000000 (Section 4). Fourth, in the third tool every angle is doubled. Two orthogonal polarizations stand at 90 degrees as Jones vectors and at 180 as Stokes vectors — orthogonal turned out to mean not “perpendicular” but “opposite” (Section 5). Fifth, a depolarizer has no Jones matrix. Passing light through five random Jones matrices gives a degree of polarization of 1 every time — the second tool can write an operation the first cannot (Section 6).

Scope of this paper (scope note): NO NEW MATHEMATICAL THEOREM AND NO NEW PHYSICAL LAW IS CLAIMED — the Jones calculus, the Stokes parameters, Mueller matrices and the Poincaré sphere are all standard. The formalism of quantum states is not derived — that the description of polarization has the same shape as a density matrix is a standard fact, and the distinct roots of pure and mixed belong to Papers 162 and 138. Nothing is claimed about any real optical element — every number comes from idealised matrices and vectors. The theory of partial coherence is not treated — what is compared here is only a fully coherent sum and a fully incoherent one. On the name: the corpus already carries “Stokes” in 18 papers, and every one of them is either Stokes's theorem (Papers 92, 104, 147) or the Navier–Stokes equations (Papers 03, 130, 196), with the Stokes parameters never once appearing — one man's name arriving as a third and different tool. What is added is counting that 200000 Jones vectors return a degree of 1 without exception, showing that the same two beams split into 1 and 0 by the manner of addition, measuring weight by weight that mixtures fill the interior, and confirming across four pairs that the angles are doubled.

1. Notation and Position

Polarization is written in three ways. (a) **The Jones vector**: a two-component complex vector. (b) **The Stokes parameters**: four real numbers (S_0, S_1, S_2, S_3) , each measurable as **an intensity**. (c) **The Poincaré sphere**: the geometry obtained by reading the last three as a point in space. The **degree of polarization** is $\sqrt{S_1^2 + S_2^2 + S_3^2}/S_0$. Every number below is computed here.

Position: The corpus has used the name “Stokes” in 18 papers, and in every one of them it was a theorem or an equation. This paper treats a third tool of that name — and what matters is not that there are three tools but that there is something the first cannot write.

2. A Jones Vector Returns a Degree of 1 and Nothing Else

Check 2.1: make 200000 random complex two-component vectors and measure the degree of polarization of each.

Quantity measured	Value
Smallest degree	0.9999999999999999
Largest degree	1.0000000000000001
Mean	1.0000000000000000
Largest departure from 1	6.661×10^{-16}

Observation 2.2

Not one exception. This is not a case of the light happening to be polarized: it is that the quantity cannot take any other value. Inside the Jones language there is nowhere to put the question “how polarized is it”.

3. The Core: the Same Two Beams Give 1 and 0 by the Manner of Addition

Check 3.1: combine equal horizontal and vertical beams in two ways.

Manner of combining	Stokes vector	Degree
Added as amplitudes (coherently)	(1, 0, 1, 0)	1.0000000000000000
Added as intensities (incoherently)	(2, 0, 0, 0)	0.0000000000000000

Observation 3.2

They are the same two beams. Added as amplitudes they give light polarized at 45 degrees; added as intensities they give unpolarized light. Both are correct — what differs is only whether the phases of the two are locked together.

Observation 3.3

And a Jones vector can write only the first. Adding vectors is adding amplitudes, and no operation corresponding to adding intensities exists. What the first tool cannot write is not an answer but a way of adding.

4. Mixtures Fill the Interior of the Ball

Check 4.1: mix horizontal and vertical as intensities, with weights w and $1 - w$.

w	Stokes vector	Degree
1.00	(1, 1, 0, 0)	1.000000
0.90	(1, 0.8, 0, 0)	0.800000
0.75	(1, 0.5, 0, 0)	0.500000
0.60	(1, 0.2, 0, 0)	0.200000
0.50	(1, 0, 0, 0)	0.000000

Observation 4.2

Fully polarized light lives on the surface of the ball and mixtures live inside it. A Jones vector can only ever reach the surface — the interior is, for the first tool, a place that does not exist.

5. In the Third Tool Every Angle Is Doubled

Check 5.1: measure the angle between two polarizations on the Jones side and on the Stokes side.

Pair	Angle as Jones vectors	Angle as Stokes vectors
Horizontal / vertical	90.0000	180.0000
+45 / −45 degrees	90.0000	180.0000
Right / left circular	90.0000	180.0000
Horizontal / +45 degrees	45.0000	90.0000

Observation 5.2

Every angle is doubled. “Orthogonal” turned out to mean, on the sphere, not perpendicular but opposite. The doubling is no accident but follows from intensity being the square of amplitude — though this paper does not derive that correspondence.

6. A Depolarizer Has No Jones Matrix

Check 6.1: on the Stokes side, a matrix with $(1, d, d, d)$ on its diagonal shrinks the ball toward its centre.

d	Stokes vector out	Degree
1.00	(1, 0, 1, 0)	1.000000
0.80	(1, 0, 0.8, 0)	0.800000
0.50	(1, 0, 0.5, 0)	0.500000
0.20	(1, 0, 0.2, 0)	0.200000
0.00	(1, 0, 0, 0)	0.000000

Check 6.2: a Jones matrix, by contrast, maps a vector to a vector, so the light coming out is always fully polarized. Five random matrices gave degrees of 1, 1, 1, 1, 1.

Observation 6.3

The second tool can write an operation the first cannot. What cannot be written is not a particular device but the very action of lowering the degree of polarization.

7. The Separator and the Fence

Observation 7.1 (what each can write)

Tool	What it can write	What it cannot
The Jones vector	only the surface of the ball	unpolarized light, depolarization
The Stokes parameters	the surface and the interior	(nothing, as regards polarization)
The Poincaré sphere	the same thing as geometry	intensity as a number

Observation 7.2 (the separator)

What separates the three is what gets added. Jones adds amplitudes and Stokes adds intensities. And since intensity is the square of amplitude, the two additions cannot agree — the 1 and the 0 of Section 3 are the size of that disagreement.

Observation 7.3 (the fence)

The fence around the Jones calculus is coherence. For light whose phase is fixed, the first tool suffices — and the moment light of unlocked phases is mixed, that language no longer has the sentence it needs. It is not that it stops working — nor that it gives a wrong answer. It is that the question can no longer be asked.

Observation 7.4

In Paper 201, on the Lorentzian side the phrase “metrically complete” stopped standing up at all — it did not become false but became unstatable. The same shape occurs here: inside the Jones calculus the degree of polarization is not false but unwritable.

8. The Census and the Close

What this paper measured:

1. Jones returns a degree of 1 only (Check 2.1): 200000 of them, departing by at most 6.66×10^{-16} .
2. The same two beams give 1 and 0 (Check 3.1).
3. Mixtures fill the interior (Check 4.1): from 1.0 down to 0.0.
4. Angles are doubled (Check 5.1): orthogonal means opposite.
5. A depolarizer has no Jones matrix (Check 6.2): all five gave 1.
6. The separator is what gets added (Observation 7.2).
7. The fence is coherence (Observation 7.3).

Close: About the same light, three tools hold three languages. One of them cannot write a single character of what the other two write as a matter of course — asked “how polarized is this light”, a Jones vector always answers “1”, and that answer is not a lie. Inside that language no other answer exists. Whether a tool is adequate cannot be settled

by checking whether the answers it gives are right — it is settled only by checking whether the question one wants to ask can be written in it at all.

References

1. Stokes, G. G. (1852). *On the Composition and Resolution of Streams of Polarized Light from Different Sources*. Transactions of the Cambridge Philosophical Society, 9.
2. Jones, R. C. (1941). *A New Calculus for the Treatment of Optical Systems*. Journal of the Optical Society of America, 31(7).
3. Mueller, H. (1948). *The Foundation of Optics*. Journal of the Optical Society of America, 38.
4. Poincaré, H. (1892). *Théorie mathématique de la lumière*. Volume 2. Paris: Gauthier-Villars.
5. Born, M., Wolf, E. (1999). *Principles of Optics*. Seventh edition. Cambridge: Cambridge University Press.
6. Yamagishi, Y. (2026). *The Equals Sign Has Distinct Roots* [Paper 16]. Zenodo. 10.5281/zenodo.21600471
7. Yamagishi, Y. (2026). *The Shared Alphabet* [Paper 92]. Zenodo. 10.5281/zenodo.21758486
8. Yamagishi, Y. (2026). *Which Part Is the Hamiltonian Is a Convention* [Paper 138]. Zenodo. 10.5281/zenodo.22081512
9. Yamagishi, Y. (2026). *The Same $\mathbf{H}^2$, and with Different Coefficients Nothing Is Quantised* [Paper 147]. Zenodo. 10.5281/zenodo.22083144
10. Yamagishi, Y. (2026). *The Whole Can Be Better Known Than Its Parts* [Paper 162]. Zenodo. 10.5281/zenodo.22092568
11. Yamagishi, Y. (2026). *“Complete” Is Four Separate Statements* [Paper 201]. Zenodo. 10.5281/zenodo.22109052
12. Yamagishi, Y. (2026). *The Spaces Converge and the Dimension Does Not* [Paper 206]. Zenodo. 10.5281/zenodo.22110637

— *From thinking to syncing.* —

Yuuki YAMAGISHI (山岸 勇輝)

© 2026 Yamagishi Complexity Torus Lab, Japan · Licensed under Creative Commons Attribution 4.0 International (CC BY 4.0)