

The Peak of a Pulse Can Leave Before It Arrives

— Four Quantities Are Called the Speed of Light in a Medium, and the Group Speed Reaches $-25.4 c$ While the Front Alone Walks at Exactly c — [Paper 210]

Yuuki YAMAGISHI (山岸 勇輝)

Yamagishi Complexity Torus Lab, Japan

August 2026

— *From thinking to syncing.* —

ABSTRACT

There is more than one quantity called the speed of light in a medium. This paper counts four of them and writes out, in numbers, which of them relativity actually fences. **NO NEW MATHEMATICAL THEOREM AND NO NEW PHYSICAL LAW IS CLAIMED.** First, a phase speed above c is nothing unusual. In a waveguide the phase and group speeds multiply to exactly c^2 , agreeing in every digit carried, and in a thin plasma the phase speed reaches $7.12 c$ at $\omega = 1.01 \omega_p$ (Section 2). Second, the first experiment failed. A pulse was fired straight at an absorption line, where the group index is -14.60 , and its peak came out late, at 63.65 against a vacuum arrival of 20 — the carrier had been cut to 1.1×10^{-4} , so what emerged was not the pulse but its wings (Section 3). Third, with a gain doublet instead, the group speed reaches $-25.44 c$. The gain over $z = 20$ is only a factor 1.1249 , so the medium is nearly transparent and strongly anomalous at once (Section 4). Fourth, this is the core. The outgoing peak stands at $t = -1.950$, which is earlier than the incoming peak at $t = 0$. That is 21.950 ahead of the vacuum arrival, exceeding the prediction $z(1 - n_g) = 20.786$ by 5.6% — but a Gaussian has no front: even at $t = -2000$ its amplitude is 1.4×10^{-11} , which is not zero (Section 5). Fifth, give the signal a front and nothing beats c . The amplitude before arrival fell as $6.0 \times 10^{-8} \rightarrow 7.7 \times 10^{-9} \rightarrow 9.8 \times 10^{-10}$ as the step was halved — what was there was the grid and not the medium (Section 6).

Scope of this paper (scope note): NO NEW MATHEMATICAL THEOREM AND NO NEW PHYSICAL LAW IS CLAIMED — the separation of signal speed from front speed by Sommerfeld and Brillouin, and negative group velocity in a gain medium, are standard and have been measured. It is not claimed that faster-than-light signalling is possible — the paper shows the opposite, watching in numbers that the front does not beat c . No definition of the speed of information is offered — what is treated is the position of the front, not the question of where information becomes readable. The energy velocity is not computed — it is a fifth quantity, and only four are counted here. No constants of any real medium are used — the units are $c = 1$ and the lines sit at $\omega = 1$ in a model. Relation to Papers 11 and 21: 11 treated the no-signalling theorem for quantum correlations and 21 a group-velocity shift from quantum gravity — both are other stories, and this paper looks only at classical electromagnetic dispersion. What is added is checking to the last digit carried that the two speeds multiply to c^2 in a waveguide, writing out the failed experiment and why it failed, locating numerically the band where the group speed is negative, measuring that the peak stands earlier than the input, measuring that with a genuine front the pre-arrival amplitude falls with the grid, and measuring the exponent with which the index returns to one.

1. Notation and Position

Units are chosen so that $c = 1$. In a dispersive medium

$$v_p = \frac{\omega}{k}, \quad v_g = \frac{d\omega}{dk}$$

are the phase and group speeds, with the group index $n_g = n + \omega dn/d\omega$ and $v_g = c/n_g$. The third quantity is the **front speed**, the speed at which the instant separating “nothing has arrived yet” from “something has” advances. The fourth is the **signal speed**.

Position: In Paper 197 “converges” turned out to be four separate claims — here “the speed of light” is four separate quantities. In Paper 207 one of three tools could not write the question — here, of four quantities, only two are fenced at all.

2. Exceeding c Is Nothing Unusual

Check 2.1: measure both speeds in a waveguide, $\omega^2 = c^2 k^2 + \omega_c^2$ with $\omega_c = 1$.

k	Phase speed	Group speed	Product
0.25	4.12310562562	0.242535625036	1.0
0.5	2.2360679775	0.4472135955	1.0
1	1.41421356237	0.707106781187	1.0
5	1.01980390272	0.980580675691	1.0

At every k the phase speed is above c , the group speed below it, and the product is one in every digit carried.

Check 2.2: a thin plasma, $n = \sqrt{1 - (\omega_p/\omega)^2}$.

ω/ω_p	Index n	Phase speed / c
1.01	0.1403707612	7.12399072
1.1	0.4165977905	2.400396793
2	0.8660254038	1.154700538
100	0.9999499987	1.000050004

Observation 2.3

The index of glass for hard X-rays is about $1 - 10^{-6}$, so the phase speed exceeds c by a part in a million every day and everywhere. Nobody is troubled by it — because a phase cannot be marked.

3. The First Experiment Failed

Looking for a place where the group speed is negative and firing a pulse into it, the first choice was **straight at an absorption line**: $f = 0.05$, $\omega_0 = 1$, $\gamma = 0.05$, carrier $\omega = 1.00$, group index -14.60 .

Check 3.1: the outgoing peak stood at $t = 63.65$, against $t = 20$ in vacuum. **Not early: more than three times late.**

ω	$\text{Im } n$	Transmission over $z = 20$
0.99	-0.3576874	8.399×10^{-4}
1.00 (carrier)	-0.4550899	1.115×10^{-4}
1.20	-0.008069653	0.8239
1.50	-0.001220259	0.9641

Observation 3.2

The medium was right and the experiment was wrong. The carrier was cut to 1.1×10^{-4} , so what survived was the far wings, which are barely absorbed — and the wings arrive late.

Observation 3.3

There is something to learn from that: a negative group speed is invisible unless the medium is nearly transparent across the whole band of the pulse. “The group speed is negative” and “the peak comes out early” are not the same statement — the first is a property of the dispersion relation, the second is about what actually survives.

4. A Gain Doublet Reaches $-25.4 c$

Put two **gain** lines of width 0.01 at $\omega = 0.98$ and 1.02 , which is what the laboratory does.

Check 4.1:

ω	$\text{Re } n - 1$	$\text{Im } n$	Group index	Group speed / c
0.90	-5.45×10^{-3}	2.80×10^{-4}	0.944481	1.0588
0.99	1.11×10^{-2}	1.10×10^{-2}	-0.410674	-2.4350
1.00	-7.16×10^{-4}	5.89×10^{-3}	-0.039305	-25.4422
1.01	-1.27×10^{-2}	1.17×10^{-2}	-0.502749	-1.9891
1.10	4.96×10^{-3}	2.81×10^{-4}	0.943470	1.0599

At the carrier the gain over the whole distance $z = 20$ is only a factor 1.1249.

Observation 4.2

Nearly transparent, and strongly anomalous. This is exactly what Section 3 lacked.

5. The Core: the Peak Leaves before It Arrives, and a Gaussian Has No Front

Check 5.1: send a Gaussian of length 400 with carrier $\omega = 1.00$ through a distance of 20.

Measured	Time
Peak of the input	0.000
Peak after vacuum	20.000
Peak after the medium	-1.950
Advance against vacuum	21.950
Prediction $z(1 - n_g)$	20.786

Observation 5.2

The outgoing peak stands earlier than the incoming one. This is real and has been measured in the laboratory. The measured advance exceeds the prediction by 5.6% — the group speed sees the dispersion only to first order, and the remainder is the higher terms.

Check 5.3: how large is the input long before its peak?

Time	Amplitude of the input
−1200	1.234×10^{-4}
−1600	1.125×10^{-7}
−2000	1.389×10^{-11}

Observation 5.4

It is not zero. A Gaussian is nonzero at every time, so the outgoing peak can be assembled entirely from the leading tail that arrived long before. Nothing was transmitted early — the medium reshaped a signal that was already there. So from the observation that the peak arrived early, nothing whatever follows about causality.

6. Give the Signal a Front, and Nothing Beats c

Into the same medium send a field that is **exactly zero** for $t < 0$. Arrival should be at $t = 20$.

Check 6.1:

Step	Largest before $t = 20$	Largest after	Ratio
0.0500	6.027×10^{-8}	1.241874	4.85×10^{-8}
0.0250	7.723×10^{-9}	1.241874	6.22×10^{-9}
0.0125	9.776×10^{-10}	1.241874	7.87×10^{-10}

Observation 6.2

Each halving of the step divides the pre-arrival amplitude by about eight. The value after arrival does not move from 1.241874 — so what was there was the grid and not the medium. The front does not beat c .

Observation 6.3

One cannot exhibit a zero numerically — all that can be exhibited is that it falls with the resolution. In Paper 209 it was a remainder of an integer that guaranteed the approach never arrives — the part played here by that remainder is played by the next section.

7. The Index Returns to One Like the Inverse Square

Check 7.1:

ω	$ \operatorname{Re} n - 1 $	$\times \omega^2$	Local slope
10	1.0105×10^{-5}	0.0010105	—
10^2	1.0005×10^{-7}	0.0010005	−2.004328
10^3	1.0004×10^{-9}	0.0010004	−2.000043
10^4	1.0004×10^{-11}	0.0010004	−2.000000

(At 10^6 the double-precision floor is reached, so it is not listed.)

Observation 7.2

The front is the rise out of “nothing has arrived yet”, so it is built from arbitrarily high frequencies. And there the index has returned to one — the medium is not there. That is why the front walks at exactly c . This is not an approximate statement: it follows from $n \rightarrow 1$ as $\omega \rightarrow \infty$.

8. The Separator and the Fence

Census 8.1: one medium, one frequency, $\omega = 1.00$.

Quantity	Value / c	Fenced?
Phase speed	1.000717	no
Group speed	−25.4422	no, it even goes negative
Front speed	1.000000	yes, exactly c
Signal speed	≤ 1	yes

Observation 8.2 (the separator)

What separates the four is what can be marked. A phase cannot be marked, a sine wave being endless and every crest the same as every other. A peak can be marked, but a peak can be rebuilt out of the leading tail. The only thing that can be marked and cannot be rebuilt is the front — the passage from “nothing yet” to “something” has nothing in front of it, so it cannot be made from what came before.

Observation 8.3 (the fence)

The fence around “nothing exceeds c ” is that the subject be the front. Outside that fence the sentence becomes false — phase and group speeds exceed c without difficulty. And forbidding those two would protect nothing: the quantity that had to be forbidden was a different one all along.

Observation 8.4

In Paper 207 the degree of polarization was, inside the Jones language, not false but unwritable — here the trouble is the opposite: too much can be written. All four are definable, all four are computable, and two of them carry no restriction at all. “Nothing exceeds the speed of light” is neither true nor false until one settles which quantity is meant.

9. The Census and the Close

What this paper measured:

1. In a waveguide the two speeds multiply to c^2 (Check 2.1).
2. A phase speed above c is ordinary (Check 2.2): **7.12 c** .

3. **The first experiment failed (Check 3.1): the peak arrived late, at 63.65.**
4. **The gain doublet gives $-25.44 c$ (Check 4.1): with a gain of only 1.1249.**
5. **The peak stands earlier than the input (Check 5.1): $t = -1.950, 21.950$ ahead of vacuum.**
6. **A Gaussian has no front (Check 5.3): 1.4×10^{-11} even at $t = -2000$.**
7. **With a front, nothing beats c (Check 6.1): falling by eight with each halving.**
8. **The index returns to one like ω^{-2} (Check 7.1): slope -2.000000 .**

Close: The peak of a pulse really does leave before it arrives. It has been measured, and here it stood at $t = -1.950$. And still nothing at all was transmitted early — because that peak was rebuilt out of the leading tail that had arrived long before. What is early is the peak, not the news. And on the side of the news, the instant at which “nothing yet” ends is built from arbitrarily high frequencies — and there the medium is not there. So the front walks at exactly c inside any medium whatever. Whether “faster than light” is true or false is not settled until one settles which quantity is being spoken of.

References

1. Sommerfeld, A. (1914). Über die Fortpflanzung des Lichtes in dispergierenden Medien. *Annalen der Physik*, 44.
2. Brillouin, L. (1914). Über die Fortpflanzung des Lichtes in dispergierenden Medien. *Annalen der Physik*, 44.
3. Brillouin, L. (1960). *Wave Propagation and Group Velocity*. New York: Academic Press.
4. Garrett, C. G. B., McCumber, D. E. (1970). Propagation of a Gaussian light pulse through an anomalous dispersion medium. *Physical Review A*, 1.
5. Chiao, R. Y. (1993). Superluminal (but causal) propagation of wave packets in transparent media with inverted atomic populations. *Physical Review A*, 48.
6. Wang, L. J., Kuzmich, A., Dogariu, A. (2000). Gain-assisted superluminal light propagation. *Nature*, 406.
7. Milonni, P. W. (2005). *Fast Light, Slow Light and Left-Handed Light*. Bristol: Institute of Physics Publishing.
8. Jackson, J. D. (1999). *Classical Electrodynamics*. Third edition. New York: Wiley.
9. Yamagishi, Y. (2026). *Factors That Cancel in Normalisation, and the Exponent 2 of Born's Rule* [Paper 11]. Zenodo. 10.5281/zenodo.21404636
10. Yamagishi, Y. (2026). *The Equals Sign Has Distinct Roots Too* [Paper 16]. Zenodo. 10.5281/zenodo.21600471
11. Yamagishi, Y. (2026). *Zero-Tension Does Not Act on an Identity* [Paper 21]. Zenodo. 10.5281/zenodo.21601143
12. Yamagishi, Y. (2026). *The One-Way Speed of Light Cannot Be Measured* [Paper 24]. Zenodo. 10.5281/zenodo.22090261
13. Yamagishi, Y. (2026). *“Converges” Is Four Separate Claims, and What Separates Them Is What You May Discard* [Paper 197]. Zenodo. 10.5281/zenodo.22107590

14. Yamagishi, Y. (2026). *There Are Three Tools for Writing Polarization, and One of Them Cannot Write Unpolarized Light* [Paper 207]. Zenodo. 10.5281/zenodo.22110902
15. Yamagishi, Y. (2026). *Equal Volume Is Not Enough to Cut and Reassemble* [Paper 209]. Zenodo. 10.5281/zenodo.22111317

— *From thinking to syncing.* —

Yuuki YAMAGISHI (山岸 勇輝)

© 2026 Yamagishi Complexity Torus Lab, Japan · Licensed under Creative Commons Attribution 4.0 International (CC BY 4.0)