

A Flat Slab Brings Light to a Focus

— With a Negative Index Light Bends to the Same Side of the Normal and a Slab of Thickness d Forms an Image $2d$ Away: the Aberration Vanishing Exactly Only When n Is Precisely Minus One — [Paper 219]

Yuuki YAMAGISHI (山岸 勇輝)

Yamagishi Complexity Torus Lab, Japan

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— *From thinking to syncing.* —

ABSTRACT

That an index of refraction is less than one is nothing unusual (Paper 210). What this paper treats is its being negative. **NO NEW MATHEMATICAL THEOREM AND NO NEW PHYSICAL LAW IS CLAIMED.** First, which square root to take is settled by passivity. For $\varepsilon = \mu = -1 + 0.01i$ the principal root is $1.0 - 0.01i$, which would amplify, and the passive one is the other, $-1.0 + 0.01i$ (Section 2). Second, of the four combinations of sign only two give a real index. If ε and μ differ in sign then n is purely imaginary and nothing propagates, and only when both are negative is n real and negative (Section 3). Third, a negative index bends light to the same side of the normal. At an incidence of 30° , $n = 1.5$ gives $+19.47^\circ$ and $n = -1.5$ gives -19.47° (Section 4). Fourth, this is the core. Rays from a point source 0.4 in front of a slab of thickness 1 all crossed the axis at 1.6 , whether they left at 2° or at 30° , when $n = -1$ — the spread is **0.0**: there is no aberration (Section 5). Fifth, that perfection stands at a single point. The spread is proportional to $|n + 1|$, being 3.31×10^{-4} at $n = -1.001$ and 1.44×10^{-2} at -1.05 — for comparison a sphere of $n = 1.5$ spreads its focus by **0.10997462** over the same angles (Sections 6 and 7).

Scope of this paper (scope note): NO NEW MATHEMATICAL THEOREM AND NO NEW PHYSICAL LAW IS CLAIMED — Veselago's analysis of media with both constants negative, Pendry's flat lens and the experimental confirmation by Shelby and others are all standard. Loss is not treated — the ray tracing uses the real part only and absorption is not included. Imaging beyond the diffraction limit, by amplification of the near field, is not treated — what appears is the focus of geometrical optics alone. The design of metamaterials is not discussed — ϵ and μ are numbers that were given. Nothing is claimed about any real material. Relation to Paper 210: 210 treated $n < 1$, in plasmas and for X-rays, and wrote that the phase speed exceeds c — what is treated here is $n < 0$, a different matter. But the conclusion of 210 stands here as well: the front travels at exactly c and the group velocity remains positive. What is added is confirming in four cases that passivity settles the branch of the square root, writing out that only two of the four sign combinations give a real index, tabulating the refracted angles for a negative index, obtaining the focus of a flat slab by ray tracing and finding the spread to be zero, measuring at six values of n that the spread is proportional to $|n + 1|$, and comparing with the aberration of an ordinary sphere.

1. Notation and Position

The index of refraction is $n = \sqrt{\epsilon\mu}$. The square root has two branches and **the formula does not choose between them**; what chooses is **that the medium be passive**, that is, that a wave **decay rather than grow**. With the convention $\exp(i(kz - \omega t))$ this means $\text{Im } n > 0$.

Position: In Paper 210 there were four speeds of light and only the front carried a fence — here the index itself turns negative and the fence on the front does not move. In Paper 212 the impossibility of laying a sphere on flat paper was written in terms of curvature — here a flat slab forms an image.

2. Passivity Chooses the Square Root

Check 2.1: give ϵ and μ a small imaginary part $0.01i$ and set the two branches side by side.

ϵ	μ	Principal root	The other	Which is passive
$2 + 0.01i$	$1 + 0.01i$	$1.414218 + 0.010607i$	$-1.414218 - 0.010607i$	principal
$-1 + 0.01i$	$1 + 0.01i$	$0 + 1.00005i$	$0 - 1.00005i$	principal
$-1 + 0.01i$	$-1 + 0.01i$	$1.0 - 0.01i$	$-1.0 + 0.01i$	the other
$-2 + 0.01i$	$-3 + 0.01i$	$2.449491 - 0.010206i$	$-2.449491 + 0.010206i$	the other

Observation 2.2

When ε and μ are both negative, the principal root makes the medium amplify. Passivity demands the negative branch, and there n is a real negative number. To write that “ $n = \sqrt{\varepsilon\mu}$, hence positive” is to have forgotten to choose a branch.

3. Of Four Sign Combinations, Two Give a Real Index

Check 3.1:

ε, μ	The passive n	What happens
+, +	$1.414218 + 0.010607i$	ordinary
-, +	$0 + 1.000050i$	no propagation (an evanescent wave)
+, -	$0 + 1.000050i$	no propagation (an evanescent wave)
-, -	$-1.0 + 0.01i$	a negative index

Observation 3.2

With only one of them negative the light cannot get in — which is why metals do not transmit. Only when both are negative does light enter, and bend the wrong way.

4. It Bends to the Same Side of the Normal

Check 4.1: the refracted angle for an incidence of 30° .

n	Refracted angle	Side of the normal
1.5	$+19.471221^\circ$	the far side
1.0	$+30^\circ$	the far side
0.5	$+90^\circ$	the far side (grazing)
-1.0	-30°	the same side
-1.5	-19.471221°	the same side

Observation 4.2

The size is set by the magnitude of n and the sign sets the direction. $n = 1.5$ and $n = -1.5$ give the same angle on opposite sides — and nothing else in optics does this.

5. The Core: A Flat Slab Brings Light to a Focus

Put a point source at a distance $s = 0.4$ in front of a slab of thickness $d = 1$. Rays bend to **the same side** at the entrance, converge inside, and resume their original angle at the exit.

Check 5.1: where rays leaving between 2° and 30° cross the axis.

n	First ray	Last ray	Spread
-1	1.6	1.6	0.0
-1.05	1.55232696943	1.53795809922	0.0143689
-1.2	1.43317812137	1.39388418604	0.0392939
-0.9	1.71127005977	1.75727512472	0.0460051

(The ideal focus at $n = -1$ is $2d - s = 1.6$.)

Observation 5.2

At $n = -1$ every ray, at every angle, crosses at the same point to the last digit carried. Without a single curved surface, a flat slab alone makes an image — and there is no aberration: the spherical aberration that every ordinary lens carries is entirely absent.

Observation 5.3

The reason is simple. At $n = -1$ a ray travels inside the slab at the negative of its entry angle and resumes it on leaving, so the thickness acts as a negative distance — and the image position $2d - s$ does not depend on the angle.

6. That Perfection Stands at a Single Point

Check 6.1:

n	Spread	Spread $\div n + 1 $
-1	0.0	—
-1.001	3.311192×10^{-4}	0.33111917
-1.01	3.223652×10^{-3}	0.32236525
-1.05	1.436887×10^{-2}	0.28737740
-1.5	5.406852×10^{-2}	0.10813704

Observation 6.2

The spread is proportional to $|n + 1|$, with a constant tending to $1/3$ as $n \rightarrow -1$. So the perfect focus is a single point in the space of materials and not a region — a departure of one part in a thousand already gives an aberration of 3.3×10^{-4} .

7. Compared with an Ordinary Curved Surface

Check 7.1: parallel rays at height h onto a sphere of radius 1 with $n = 1.5$.

Height h	Focus
0.02	2.999733313
0.1	2.993320313
0.2	2.973122217
0.4	2.889758695

The spread is **0.10997462**.

Observation 7.2

An ordinary material with a curved face spreads its focus by 0.11, and the flat slab at $n = -1$ does not spread it by a digit. This is a comparison within geometrical optics: neither loss nor diffraction is included.

8. What Does Not Bend, and the Separator and the Fence

Census 8.1: inside a medium with $n < 0$, what turns negative and what does not.

Quantity	Sign
Phase velocity	negative
Group velocity	positive
Flow of energy	positive
Speed of the front	exactly c

Observation 8.2

The crests travel backwards while the energy travels forwards. And the front overtakes nothing — a negative index holds only over a band, since the medium must be dispersive, and at high frequency n returns to one (the same reason as in Paper 210).

Observation 8.3 (the separator)

What separates the cases of “index” is the pair of signs of ϵ and μ : alike gives a real n , unlike gives a purely imaginary one and nothing propagates. And among the alike, only both negative makes the sign of n negative — and what settles that is not the formula but passivity.

Observation 8.4 (the fence)

The fence around “a flat slab makes no image” is that $n > 0$. Outside it the sentence is false — and the fence around “an image without aberration” is narrower still: the single point $n = -1$. In Paper 217, the moment the fence (surjectivity) was dropped, a whole function's worth of freedom appeared — here, on the contrary, the inside of the fence is a single point.

9. The Census and the Close

What this paper measured:

1. Passivity chooses the branch (Check 2.1): both negative gives the negative branch.
2. Unlike signs do not propagate (Check 3.1): n purely imaginary.
3. It bends to the same side (Check 4.1): -19.47° against 30° .
4. A flat slab focuses (Check 5.1): spread 0.0, focus $1.6 = 2d - s$.
5. The perfection is a single point (Check 6.1): spread $\propto |n + 1|$, coefficient $1/3$.
6. An ordinary sphere spreads by 0.11 (Check 7.1).

Close: A lens is curved because it could not gather light if it were not. Make the index negative, however, and a flat slab brings light to a focus — and carries not one of the aberrations that every ordinary lens must have. But only where n is exactly -1 : a departure of one part in a thousand already shows. The perfect instrument stands at a single point in the space of materials. And what turned negative was only the direction the phase travels, the energy

and the front still going forward — which of the two roots of the one formula $n = \sqrt{\epsilon\mu}$ to take was settled not by the formula but by “it must not amplify”.

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Yuuki YAMAGISHI (山岸 勇輝)

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