

It Passes Every Test, and the Points Lie on Fifteen Planes

— Random Has Distinct Roots, and Neither Statistical Tests nor Compression Detects RANDU: Taken Three at a Time, Sixty Thousand Points Line Up on Just Fifteen Parallel Planes — [Paper 223]

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— *From thinking to syncing.* —

ABSTRACT

“It is random” carries two separate meanings: that it passes tests and that it cannot be written briefly. This paper exhibits in numbers a sequence that passes all of the first and breaks the second spectacularly. NO NEW MATHEMATICAL THEOREM AND NO NEW LAW IS CLAIMED. First, it passes every test. RANDU has a mean of 0.501293, a runs statistic of -0.543 , a serial correlation of -0.000029 and a chi-square on bytes of 256.9 against an expected 255 — indistinguishable from a modern generator (Section 2). Second, compression does not detect it either. Under zlib, RANDU comes to 0.9993 of its size against 1.0004 for a modern generator, all but the same (Section 3). Third, taken two at a time almost nothing shows. Searching small relations between consecutive pairs gives 25353 distinct values for RANDU against 59878 for the comparison — a faint trace, and not enough to raise an alarm (Section 4). Fourth, this is the core. Taken three at a time, $x_{i+2} - 6x_{i+1} + 9x_i$ takes exactly 15 values, and they are the whole numbers from -5 to 9 , with a departure from a whole number of 0 — sixty thousand points lying on fifteen parallel planes (Section 5). Fifth, the information actually held is 70 bytes. The procedure that makes 60002 bytes is 66 bytes plus a seed of 4, a ratio of 0.00117 — a thousandth of the 0.9993 that the best compressor reached (Section 6).

Scope of this paper (scope note): NO NEW MATHEMATICAL THEOREM AND NO NEW LAW IS CLAIMED — Marsaglia's “random numbers fall mainly in the planes”, Kolmogorov complexity and its uncomputability are all standard. No generator is designed — only two old ones and one modern one appear. No survey of tests is offered — four tests and one compressor are used, and that some test would catch it is of course true (a test aimed at lattice structure finds it at once). Kolmogorov complexity is not computed — only an upper bound can be. Cryptographic security is not treated. Relation to Paper 220: there, all three pairs were independent and the three together were not — RANDU is likewise clean in pairs and broken in triples. One shape appearing in two places, probability and the generation of numbers. And 220 wrote that a search can rule out small relations and never settle the question — Section 7 here is the general form of that asymmetry. What is added is measuring that four tests and a compressor fail to separate three generators, measuring that pairs show almost no trace, showing that the triples take exactly fifteen whole-number values, and giving the length of the short procedure against what compression achieved.

1. Notation and Position

There are at least four senses of “random”: **passing tests** (statistical), **not being writable briefly** (Kolmogorov), **not being predictable**, and **having come from such a mechanism** (a statement about a process). This paper compares **the first two**.

The generators used are RANDU, $x_{i+1} = 65539 x_i \bmod 2^{31}$; a comparison, $x_{i+1} = (1103515245 x_i + 12345) \bmod 2^{31}$; and a modern generator.

Position: In Paper 220 all three pairs were independent and the three together were dependent — here too it is clean in pairs and broken in triples. In Paper 221 passing the test of being diagonalisable guaranteed nothing — the same holds here.

2. It Passes Every Test

Check 2.1: generate 60002 values and apply four tests.

Generator	Mean	Runs, z	Serial correlation	Chi-square on bytes
RANDU	0.501293	−0.543	−0.000029	256.9
Comparison	0.500159	−0.143	−0.000639	256.0
A modern generator	0.500654	—	0.000053	—

(The expectation of a chi-square on 255 degrees of freedom is 255.)

Observation 2.2

Not one column separates the three. RANDU is the most notorious generator there is, and it passes all four.

3. Compression Does Not Detect It

Check 3.1: write the values as bytes and apply zlib.

Generator	Raw size	After zlib	Ratio
RANDU	60002	59961	0.9993
Comparison	60002	60028	1.0004
A modern generator	60002	60028	1.0004

Observation 3.2

None of them shrinks. Compression is a test like any other and misses in the same way — “incompressible, therefore random” says no more than that this compressor sees no structure.

4. Taken Two at a Time, Almost Nothing Shows

Check 4.1: search exhaustively for relations with small coefficients between consecutive pairs.

Generator	Fewest distinct values	Coefficient
RANDU	25353	$c_1 = -3$
Comparison	59878	$c_1 = 1$

Observation 4.2

RANDU is the lower of the two (25353 against 59878), and that is a faint trace and not a collapse. Both have tens of thousands of values, and so long as one looks two at a time no alarm sounds.

5. The Core: Taken Three at a Time, Fifteen Planes

RANDU satisfies $x_{i+2} = 6x_{i+1} - 9x_i \pmod{2^{31}}$ exactly. Hence, once normalised to lie between 0 and 1, the combination $x_{i+2} - 6x_{i+1} + 9x_i$ can only be a whole number.

Check 5.1:

Measured	Value
Number of triples	60000
Distinct values of $x_{i+2} - 6x_{i+1} + 9x_i$	15
Which values	$-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9$
Largest departure from a whole number	0
The same search on the comparison	59874 values

Observation 5.2

All sixty thousand points lie on fifteen parallel planes. Placed in three-dimensional space, the points do not fill it — and not one of the tests of Sections 2 to 4 noticed.

Observation 5.3

This is the shape of Paper 220. There, all three pairs were independent and the three together dependent; here, clean in pairs and broken in triples — one way of slipping through appearing in two different places.

6. The Information Actually Held Is Seventy Bytes

Check 6.1:

Measured	Value
Bytes produced	60002
The procedure that makes them	66 bytes (plus a seed of 4)
Ratio	0.00117
The ratio zlib achieved	0.9993

Observation 6.2

The shortest description is a thousandth of the length the best general compressor reached. A short description certainly exists — and no test found it.

7. The Asymmetry That Cannot Be Crossed

Observation 7.1

exhibiting a short procedure gives an **upper** bound on the complexity. A **lower** bound is another matter: Kolmogorov complexity is **uncomputable** (Chaitin).

What can be shown	How
That a short description exists	exhibit one (done in Section 6)
That a short description does not	impossible in general

Observation 7.2

So a test can say “this can be written briefly” and can never say “this cannot”. Passing carries only the meaning “no structure has been found yet” — the general form of what Paper 220 wrote, that a search rules out small relations and never settles the question.

8. The Separator and the Fence

Observation 8.1 (the separator)

What separates the four senses of “random” is random with respect to whom.

Phrase	With respect to what
Passes tests	that list of tests
Cannot be compressed	that compressor
Cannot be written briefly	every procedure (uncomputable)
Came from such a mechanism	a statement about a process, not about the sequence

Observation 8.2 (the fence)

The fence around “this sequence is random” is saying with respect to what. Outside it the sentence is not even false but empty — the same shape as “independence was checked” in 220 and “the blackbody peaks here” in 222.

Observation 8.3

And the most widely used tests are the ones that see least. Mean, runs, correlation, chi-square and compression all look at properties of one or two dimensions — and the defect of RANDU first appears in three. In 220 the correlation sat among the fourteen of twenty that held: the easier a condition is to check, the harder it is to break.

9. The Census and the Close

What this paper measured:

1. It passes every test (Check 2.1): a runs statistic of **−0.543**.
2. Compression does not detect it (Check 3.1): **0.9993** against **1.0004**.

3. Pairs show only a trace (Check 4.1): **25353** against **59878**.
4. Triples give fifteen planes (Check 5.1): **the whole numbers —5 to 9, departure 0**.
5. The information is seventy bytes (Check 6.1): **a ratio of 0.00117**.
6. Upper bounds can be shown and lower bounds cannot (Observation 7.1).

Close: The mean is a half, the runs come out right, the correlation is all but zero, the bytes are evenly spread, and compressing it gains nothing. Every test answers “random”. And the moment the numbers are taken three at a time and set in space, all sixty thousand points fall into line on fifteen planes — and the whole of it can be written in seventy bytes. The tests did not overlook it: a test sees only what it was built to see. “It is random” can be said only once one has named the adversary, and without one that sentence has no content. And while structure can be shown to be there, it can never be shown not to be — the news that a test was passed is news about our instruments and not about the sequence.

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