

The Same Inequality Sign Points at a Wall You Can Touch and at One You Cannot

— A Census of Whether Equality Is Reached: What Separates Them Is Not the Symbol but Whether the Extremiser Lies Inside the Set — [Paper 227]

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— *From thinking to syncing.* —

ABSTRACT

When a bound in physics is written down, $\leq$ and $<$ are distinguished with care. And the symbol on the page does not settle whether the wall can be touched. This paper measures, for eleven bounds, whether the equality is reached. **NO NEW MATHEMATICAL THEOREM AND NO NEW LAW IS CLAIMED.** First, some walls cannot be touched. Approaching the Buchdahl bound of $8/9$, the central pressure diverges, passing 166667 and then 1.67×10^6 times ρc^2 — the configuration that would touch the wall is not a configuration at all (Section 3). Second, some walls can be. A gaussian meets $\Delta x \Delta p = \hbar/2$ exactly to fifteen digits — the ratio measured 1.000000000000000 (Section 4). Third, my own guess was wrong. What forbids the algebraic CHSH ceiling of 4 is not relativity — a PR box reaches 4 exactly and its signalling measures 0 (Section 5). Fourth, this is the core. The number $2\sqrt{2} = 2.828427124746$ is a wall that is touched, a wall out of reach, and not a wall at all — what changed is not the number but which theory one stands inside (Section 5). Fifth, there are three separate reasons for not reaching. A divergence is required (Buchdahl); the extremiser sits at an endpoint (Landauer is still 0.0005% above at 10^6 units of time); something other than the inequality excludes it (Chandrasekhar) (Section 7).

Scope of this paper (scope note): NO NEW MATHEMATICAL THEOREM AND NO NEW LAW IS CLAIMED — the Buchdahl bound, the Tsirelson bound, PR boxes, minimum-uncertainty states, the Landauer bound and the Chandrasekhar mass are all standard. No bound is derived — for a given bound, only whether the equality is reached is measured. No claim of completeness — eleven examples appear, and the classification is not asserted to be exhaustive. No general mathematics is offered — whether a supremum is a maximum is a standard distinction in analysis, and this paper only applies it to bounds in physics and counts. No rule about notation is proposed — whether one ought to write $\leq$ or $<$ is not discussed. Relation to earlier papers: Paper 16 showed that the equals sign has six distinct roots — this paper takes the inequality side, where one symbol points at two kinds of wall. Paper 21 wrote that whether something is an identity or a hypothesis depends on which theory one stands inside — Section 5 here is the quantitative version. Paper 194 showed there is only one equality fence, and Paper 160 counted three ceilings over one inequality — this paper sorts those ceilings by whether they are touched. What is added is following the Buchdahl central pressure numerically into the limit, confirming minimum uncertainty to fifteen digits, computing the CHSH value and the signalling of a PR box, following the Landauer excess out to 10^6 units of time, and tabulating eleven bounds by whether the equality is reached.

1. Notation and Position

For a bound $X \leq B$, the equality is **reached** when some configuration realises B , and **not reached** when X comes arbitrarily close to B without ever taking it. In mathematics this is the standard distinction between a **supremum** and a **maximum**. This paper **applies it to bounds in physics and counts**.

Position: Paper 16 showed six distinct roots of the equals sign — this paper takes the inequality side. The phrase which theory one stands inside, from Paper 21, does the work in Section 5.

2. The Symbol Does Not Settle Whether the Wall Can Be Touched

Observation 2.1

there are walls written with $\leq$ that are never touched, and walls written with $\geq$ that are met exactly. **The symbol says that the bound cannot be passed; it does not say that it can be met.**

And which of the two it is cannot be read off the symbol. It has to be computed.

3. A Wall Not Touched: The Extremiser Requires a Divergence

Check 3.1: for a uniform-density interior Schwarzschild solution, measure the central pressure against the compactness $u = 2GM/(Rc^2)$.

u	$u \div (8/9)$	Central pressure $\div \rho c^2$
0.5	0.5625	0.261204
0.86	0.9675	5.10897
0.888	0.999	166.666
0.888888	0.999999	166667
0.8888888	0.9999999	1.66667×10^6

Observation 3.2

The central pressure diverges at 8/9. The configuration that would touch the wall is not a configuration — in form it is written as at most, and in physics it is strictly less than.

4. A Wall Touched: A State Meets the Equality Condition

Check 4.1: measure $\Delta x \Delta p$ for minimum-uncertainty gaussians.

Width	$\Delta x \Delta p$	$\div (\hbar/2)$
10^{-11}	$5.272859085000000 \times 10^{-35}$	1.000000000000000
10^{-10}	$5.272859085000000 \times 10^{-35}$	1.000000000000000
10^{-9}	$5.272859085000000 \times 10^{-35}$	1.000000000000000

Observation 4.2

Equality holds exactly, to fifteen digits. The equality condition of Cauchy–Schwarz is that the two vectors be proportional, and states meeting it exist — not one but a whole family.

Check 4.3: for the arithmetic and geometric means, equality holds only when all the numbers agree. Drawing **200000** triples uniformly from **0.5** to **4**, equality occurred **0 times**. At $x = (2, 2, 2)$ the difference is exactly **0**.

Observation 4.4

It is reached, and the set that reaches it has probability zero. Being able to touch a wall and touching it by chance are different things.

5. The Core: One Ceiling Changes Standing With the Ground One Stands On

Check 5.1 (the guess that failed): in a first draft I wrote that the algebraic CHSH ceiling of **4** is forbidden by no-signalling. **A PR box was built and measured.**

Measured	Value
CHSH value of a PR box	4.000000000000
Its largest signalling	0

Observation 5.2

The guess was wrong. A PR box signals not at all and reaches **4** exactly — what forbids **4** is quantum mechanics, and not relativity.

Check 5.3: the three ceilings, counted from each standpoint. The quantum maximum measured **2.828427124746** against $2\sqrt{2} = 2.828427124746$.

Standing inside	2	$2\sqrt{2}$	4
Local hidden variables	reached	impossible	impossible
Quantum mechanics	reached	reached	impossible
Any no-signalling theory	reached	reached	reached

Observation 5.4

The number $2\sqrt{2}$ is a wall that is touched, a wall out of reach, and not a wall at all. The number never changed — what changed is which theory one stands inside. This is the quantitative version of what Paper 21 wrote about identities and hypotheses.

6. A Wall Not Touched: The Extremiser Sits at an Endpoint

Check 6.1: measure the work of erasure at 300 K as the time is lengthened. The floor is $k_B T \ln 2 = 2.870978885 \times 10^{-21}$ J.

Time (units)	Work	Excess over the floor
1	$1.722587331 \times 10^{-20}$	500%
100	$3.014527829 \times 10^{-21}$	5%
10^4	$2.872414375 \times 10^{-21}$	0.05%
10^6	$2.870993240 \times 10^{-21}$	0.0005%

Observation 6.2

Every finite time lies strictly above. The floor is the limit of the set and not a member of it — written with $\geq$, and the equality never occurs.

7. The Census: Eleven Bounds

Check 7.1:

Bound	Written	Reached	Why
Heisenberg $\hbar/2$	$\geq$	yes	a state meets the equality condition
AM–GM	$\geq$	yes	one point only, of probability zero
Tsirelson $2\sqrt{2}$	$\leq$	yes	inside quantum mechanics
CHSH's 2	$\leq$	yes	inside local hidden variables
CHSH's 4	$\leq$	yes	inside no-signalling (a PR box)
CHSH's 4, standing in quantum	$\leq$	no	quantum mechanics forbids it
Schwarzschild radius	$=$	yes	the horizon is a real surface
Buchdahl $8/9$	$\leq$	no	the extremiser needs a divergence
Landauer $k_B T \ln 2$	$\geq$	no	an endpoint at infinite time
Chandrasekhar mass	$<$	no	stability excludes it, not the inequality
Speed of light, massive body	$<$	no	it would take infinite energy

Observation 7.2

The reasons for not reaching fall into three.

Reason	Example
(i) The extremiser requires a divergence	Buchdahl, the speed of light
(ii) The extremiser sits at an endpoint outside the set	Landauer
(iii) Something other than the inequality excludes it	Chandrasekhar (stability)

Observation 7.3

There is only one reason for reaching — (iv) a configuration meeting the equality condition lies inside the permitted set.

8. The Separator and the Fence

Observation 8.1 (the separator)

What separates the two kinds of $\leq$ is whether the extremiser lies inside the set. It is not the symbol.

Observation 8.2 (the fence)

The fence around this quantity is bounded by B is over which set the bound is taken. Outside it the same B becomes reachable or unreachable — the same shape as this sequence is random in Paper 223 and volume is conserved in Paper 225.

Observation 8.3

The difference between strictly less and at most is not a question of notation but a question of measurement. Writing the equality is not an assertion but a way of putting the question — the form in which one asks whether this wall can be touched.

Observation 8.4

Paper 16 counted six distinct roots of the equals sign. What this paper counted is two kinds of inequality sign — the sorting of the symbols is not finished.

9. The Census and the Close

What this paper measured:

1. Buchdahl is not touched (Check 3.1): the pressure diverges past 1.67×10^6 .
2. Heisenberg is touched (Check 4.1): the ratio is 1 to fifteen digits.
3. AM–GM is touched, with probability zero (Check 4.3): 0 hits in 200000.
4. The guess failed (Check 5.1): a PR box reaches 4 and signals 0.
5. One ceiling changes standing (Check 5.3): $2\sqrt{2}$ has three meanings.
6. Landauer is an endpoint (Check 6.1): still 0.0005% above at 10^6 .

7. Three reasons for not reaching (Observation 7.2): divergence, endpoint, outside exclusion.

Close: When bounds are written, $\leq$ and $<$ are chosen with care. And that choice does not say whether the wall can be touched. Buchdahl's $8/9$ is written as at most, and no star touches it — try, and the pressure at the centre becomes infinite. Landauer's floor is written as at least, and no finite time touches it — after a million times as long, it is still 0.0005% above. A gaussian, meanwhile, meets $\hbar/2$ exactly, to fifteen digits. And $2\sqrt{2}$ holds three different standings depending on where one stands — out of reach inside local hidden variables, touched inside quantum mechanics, and not a wall at all inside a no-signalling theory. The number never moved. So writing the equality is not a claim but a way of putting a question — the form in which one asks whether this wall can be touched. And the answer cannot be read off the symbol. One has to build the extremiser and see whether it lies inside the set that was allowed.

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