

# There Are Eight Plus Signs, and Seven Become Real Addition in the Right Coordinate

— What Stops the Eighth Is the Failure to Commute, and the Gap Is Exactly Half the Commutator — [Paper 228]

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— *From thinking to syncing.* —

## ABSTRACT

One plus sign is worn by at least eight different operations. This paper counts them and separates in numbers which are really addition and which are not. **NO NEW MATHEMATICAL THEOREM AND NO NEW LAW IS CLAIMED.** First, the composition of velocities was never a broken addition. In rapidity it is ordinary addition, and over 200000 random pairs the largest gap was  $1.887 \times 10^{-15}$  (Section 2). Second, it satisfies every group axiom. Commutativity to **0**, associativity to  $1.577 \times 10^{-14}$ , an identity to **0**, inverses to **0**, and closure in  $(-c, c)$  (Section 3). Third, the same move works elsewhere. Resistances in parallel return to addition in conductance, products in the logarithm, and the joining of quantum systems in the number of qubits — dimensions multiply and qubits add, with a gap of **0** (Sections 4 and 5). Fourth, for amplitudes the plus is genuine from the start.  $|a|^2 + |b|^2$  is always **2** while  $|a + b|^2$  runs from **4** to **0** — what breaks is not the addition but that what is added is not what is measured (Section 6). Fifth, this is the core. Exactly one case admits no coordinate, and it is the failure to commute — the ratio of the gap to half the commutator converged as  $1.000834 \rightarrow 1.000008 \rightarrow 1.000000$  (Section 7).

Scope of this paper (scope note): NO NEW MATHEMATICAL THEOREM AND NO NEW LAW IS CLAIMED — the velocity composition law, rapidity, the Baker–Campbell–Hausdorff formula, the dimension of a tensor product and interference are all standard. No completeness is claimed — eight examples appear, and they are not said to exhaust the matter. Real addition is not newly defined — it means satisfying the axioms of an abelian group, in the standard sense. No general test for the existence of such a coordinate is offered — for each example one is exhibited, or declared absent. This is not a textbook on group theory. Relation to earlier papers: Paper 16 counted six distinct roots of the equals sign and Paper 227 two kinds of inequality sign — this paper takes the arithmetic operations, continuing the sorting of the symbols. Paper 181 showed that multiplication does not decide the second system — Section 5 here is its counterpart on the addition side. Paper 222 showed that the answer moves with what one counts per — the core here has the same shape: which coordinate one adds in settles the standing of the plus. What is added is checking the group axioms of velocity composition over 200000 pairs, measuring the agreement with rapidity to  $10^{-15}$ , confirming that a product of dimensions returns to a sum in the number of qubits, measuring the associativity of amplitude addition, and verifying the half-commutator prediction across four decades of scale.

## 1. Notation and Position

By **real addition** this paper means a binary operation satisfying **the axioms of an abelian group** — commutativity, associativity, an identity, inverses, and closure. This is **the standard sense**. The question is whether an operation **written with a plus sign** satisfies them.

**Position:** Paper 16 counted six roots of the equals sign and Paper 227 two kinds of inequality — this paper takes the arithmetic side. The question of Paper 222, what one counts per, becomes here which coordinate one adds in.

## 2. Velocities: One Symbol, Three Answers

Check 2.1:

| $u, v$           | Galilean $u + v$ | Relativistic   | Through rapidity |
|------------------|------------------|----------------|------------------|
| $0.4c, 0.4c$     | 0.800000000      | 0.689655172414 | 0.689655172414   |
| $0.9c, 0.9c$     | 1.800000000      | 0.994475138122 | 0.994475138122   |
| $0.99c, 0.99c$   | 1.980000000      | 0.999949497500 | 0.999949497500   |
| $0.999c, 0.999c$ | 1.998000000      | 0.999999499500 | 0.999999499500   |

Over 200000 random pairs the largest gap between the two right-hand columns was  $1.887 \times 10^{-15}$ .

Observation 2.2

In the rapidity  $\theta = \operatorname{arctanh}(v/c)$  the composition is ordinary addition. What was broken was not the plus but the coordinate.

3. And It Satisfies Every Group Axiom

Check 3.1: for  $u \oplus v = (u + v)/(1 + uv)$ , measured over 200000 triples.

| Axiom                | Largest gap             |
|----------------------|-------------------------|
| Commutativity        | 0                       |
| Associativity        | $1.577 \times 10^{-14}$ |
| Identity (0)         | 0                       |
| Inverse ( $-u$ )     | 0                       |
| Closure in $(-c, c)$ | holds                   |

Observation 3.2

Every axiom holds. This is a real addition written in a different variable — so the phrase addition breaks down in relativity is not accurate.

4. The Same Move, Twice More

Check 4.1:

| What is added           | The coordinate    | Check                                                   |
|-------------------------|-------------------|---------------------------------------------------------|
| Resistances in parallel | Conductance $1/R$ | $1/R_1 + 1/R_2 = 0.101000000000 = 1/R_{\parallel}$      |
| Multiplication          | The logarithm     | $7 \times 11 = 77$ , $\exp(\ln 7 + \ln 11) = 77.000000$ |
| Velocities              | Rapidity          | Section 2                                               |

### Observation 4.2

The reciprocal, the logarithm, the inverse hyperbolic tangent — each turns a stubborn combination into ordinary addition.

## 5. Joining Two Systems: Dimensions Multiply, Qubits Add

Check 5.1:

| $\dim A$ | $\dim B$ | Classical (adds) | Quantum (multiplies) | Qubits (add)                     |
|----------|----------|------------------|----------------------|----------------------------------|
| 2        | 3        | 5                | 6                    | $1.000000 + 1.584963 = 2.584963$ |
| 10       | 10       | 20               | 100                  | $3.321928 + 3.321928 = 6.643856$ |
| 1024     | 1024     | 2048             | <b>1048576</b>       | $10 + 10 = 20$                   |

The gap between the sum of the logarithms and the logarithm of the product was **0**.

### Observation 5.2

Under one phrase, joining two systems, the classical world adds dimensions and the quantum world multiplies them. The coordinate that restores addition is the number of qubits — and that single line is the whole reason a quantum register is exponentially large. It is the counterpart on the addition side of Paper 181.

## 6. Amplitudes: Here the Plus Was Genuine All Along

Check 6.1:

| Phase    | $ a ^2 +  b ^2$ | $ a + b ^2$ |
|----------|-----------------|-------------|
| 0        | 2.000000        | 4.000000    |
| $\pi/3$  | 2.000000        | 3.000000    |
| $\pi/2$  | 2.000000        | 2.000000    |
| $2\pi/3$ | 2.000000        | 1.000000    |
| $\pi$    | 2.000000        | 0.000000    |

The associativity of amplitude addition was measured over **200000** triples: the largest gap was  $1.776 \times 10^{-15}$ .

**Observation 6.2**

Nothing at all is wrong with the plus here. Addition of complex numbers is entirely ordinary — what differs is that what is added is not what is measured. Interference is the gap between adding and then measuring and measuring and then adding.

## 7. The Core: The One Case No Coordinate Rescues

**Check 7.1:** the commuting case first. For  $C_1 = \text{diag}(1, 2)$  and  $C_2 = \text{diag}(3, 5)$  the commutator is **0** and the gap between  $e^{C_1}e^{C_2}$  and  $e^{C_1+C_2}$  is  $2.274 \times 10^{-13}$ , which is numerical only.

**Check 7.2:** the non-commuting case. With  $A$  and  $B$  the off-diagonal elementary matrices the commutator has largest entry 1 and the gap between  $e^Ae^B$  and  $e^{A+B}$  is **0.543081**. Scaling both by  $t$ :

| $t$       | Measured gap                 | $t^2 [A, B] /2$              | Ratio           |
|-----------|------------------------------|------------------------------|-----------------|
| $10^{-1}$ | $5.004168056 \times 10^{-3}$ | $5.000000000 \times 10^{-3}$ | 1.000834        |
| $10^{-2}$ | $5.000041667 \times 10^{-5}$ | $5.000000000 \times 10^{-5}$ | 1.000008        |
| $10^{-3}$ | $5.000000418 \times 10^{-7}$ | $5.000000000 \times 10^{-7}$ | <b>1.000000</b> |
| $10^{-4}$ | $4.999999970 \times 10^{-9}$ | $5.000000000 \times 10^{-9}$ | <b>1.000000</b> |

**Observation 7.3**

To leading order the gap is exactly half the commutator. This is not a bad coordinate — it is the failure to commute itself, and no relabelling removes it.

**8. The Census and the Separator**

Check 8.1:

| What is added                     | Coordinate restoring + | A real plus?                     |
|-----------------------------------|------------------------|----------------------------------|
| Counts                            | none needed            | yes                              |
| Velocities                        | rapidity               | yes, after a change              |
| Resistances in parallel           | conductance            | yes, after a change              |
| Factors in a product              | the logarithm          | yes, after a change              |
| Quantum systems                   | the number of qubits   | yes, after a change              |
| Probabilities of exclusive events | none needed            | yes                              |
| Amplitudes                        | none needed            | yes — but the square is measured |
| Non-commuting exponents           | none exists            | no                               |

**Observation 8.2 (the separator)**

What separates the senses of plus is whether a coordinate exists that returns it to real addition. Of the eight, seven have one and one does not — and what settles the absence is the failure to commute.

**Observation 8.3 (the fence)**

The fence around these can be added is having settled which variable they are added in. Outside it the sentence is not even false but empty — the same shape as this quantity is bounded in Paper 227 and the blackbody peaks here in Paper 222.

#### Observation 8.4

Both addition breaks in relativity and probabilities do not add in quantum mechanics are inaccurate. The first is a different coordinate and the second is a difference between what is added and what is measured — what truly breaks is the non-commuting case alone.

## 9. The Census and the Close

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What this paper measured:

1. Velocities add in rapidity (Check 2.1): a gap of  $1.887 \times 10^{-15}$ .
2. Every group axiom holds (Check 3.1): associativity to  $1.577 \times 10^{-14}$ .
3. The reciprocal and the logarithm do the same (Check 4.1).
4. Dimensions multiply, qubits add (Check 5.1): a gap of 0.
5. The plus of amplitudes is genuine (Check 6.1):  $|a + b|^2$  runs 4 to 0.
6. Half the commutator (Check 7.2): the ratio converged to 1.000000.
7. Seven of eight have a coordinate (Check 8.1).

Close: The plus sign taught in the first year of school is one symbol. The operations wearing it number at least eight. And seven of them were never broken — velocities in rapidity, parallel resistances in conductance, products in the logarithm, quantum systems in the number of qubits: every one returns to ordinary addition. Amplitudes are ordinary addition from the start — interference happens only because what is added is not what is measured. Search as one may, one case has no coordinate. When two things that do not commute are added inside an exponent — the gap is exactly half the commutator, and the measured ratio converged to 1.000000. That is not a failure of notation. The very fact that the order changes the answer is what refuses to return to addition — so before asking whether things can be added, one must ask in which variable. And only when there is no such variable are they truly not addable.

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