

There Are Seven Minus Signs, and One of Them Undoes No Addition

— A Point Minus a Point Is Not a Point: What Separates Them Is Whether It Undoes, Whether It Merely Names,
and What Type Comes Out — [Paper 230]

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— *From thinking to syncing.* —

ABSTRACT

Subtraction undoes an addition. And one thing wearing that symbol undoes no addition at all. This paper counts seven minus signs and sorts them by three questions. **NO NEW MATHEMATICAL THEOREM AND NO NEW LAW IS CLAIMED.** First, among the counting numbers it fails about half the time. Over 200000 pairs, $a - b$ stayed a counting number **50.0575%** of the time — the negative numbers were invented for this (Section 2). Second, this is the core. Set difference does not undo union. $(A \cup B) \setminus B$ came back to A in **44.0115%** of 200000 pairs, and only when the two are disjoint (counterexamples: **0**) (Section 3). Third, a point minus a point changes type. Weights summing to 1 give a point, to 0 give a vector, and anything else gives nothing at all — $P + Q$ is not unusual; it is undefined (Section 4). Fourth, infinity minus infinity depends on the road. Four expressions of the same shape returned 0, -1.667×10^{-7} , $+3.333 \times 10^{-7}$, and a divergence (Section 5). Fifth, subtracting a divergence depends on the scheme. Three ways of removing the same divergence left 0, -0.6931471806 and $+0.6931471806$ behind — differing by **$\ln 2$** (Section 6).

Scope of this paper (scope note): NO NEW MATHEMATICAL THEOREM AND NO NEW LAW IS CLAIMED — affine spaces and barycentric coordinates, set difference, indeterminate forms, the scheme dependence of renormalisation and the unary inverse are all standard. No completeness is claimed — seven appear and are not said to exhaust the matter. No theory of renormalisation is offered — the removal of the divergence is not treated, only that the finite part left behind is not settled. No correct scheme is chosen. This is not a textbook on algebra — the general theory of groups, rings and affine spaces is not treated. Relation to earlier papers: This is the fifth in the sorting of symbols, after Paper 16 (six for the equals sign), 227 (two for inequality), 228 (eight for the plus) and 229 (five for the slash). Paper 228 closed with before asking whether things can be added, ask in which variable — the answer here is its mirror image: before subtracting, ask what is supposed to come back. Paper 170 showed that cancelling the differentials gives -1 , not $+1$ — the same family, a sign hidden inside a notation. Paper 226 treated the case where the sign is the orientation itself. What is added is measuring how often the counting numbers fail, measuring how often set difference undoes union and confirming the condition with no counterexample, sorting expressions into point, vector and meaningless by the sum of the weights, comparing four indeterminate forms, and exhibiting the $\ln 2$ that separates three schemes.

1. Notation and Position

Subtraction undoes an addition: $a - b$ asks for the thing that, added to b , gives a . Three questions are asked of each case. (i) Does it really undo an addition? (ii) Is what comes out the same type as what went in? (iii) Is the answer unique?

Position: This is the fifth in the sorting of symbols. The closing line of Paper 228 turned around becomes the closing line here.

2. Among the Counting Numbers It Fails About Half the Time

Check 2.1: draw 200000 pairs of counting numbers from 0 to 999.

Measured	Value
Pairs tried	200000
$a - b$ stayed a counting number	100115
Share	50.0575%

Observation 2.2

The cure is not to adjust the operation but to enlarge the set — exactly the shape of division in Paper 229. The negative numbers were made for this and for nothing else.

3. The Core: Set Difference Does Not Undo Union

Check 3.1: let $A = \{1, 2\}$ and $B = \{2, 3\}$.

Expression	Value
$A \cup B$	$\{1, 2, 3\}$
$(A \cup B) \setminus B$	$\{1\}$
A	$\{1, 2\}$

The element **2** belonged to A , and removing B threw it out with the rest.

Check 3.2: over **200000** random pairs of small sets.

Measured	Value
$(A \cup B) \setminus B = A$ occurred	88023 times
Share	44.0115%
Disagreements with the rule “exactly when disjoint”	0 (over 20000 pairs)

Observation 3.3

Set difference is not an operation that undoes union. It undoes it only when A and B are disjoint — the very sentence subtraction undoes addition fails here. And the symbol is the same.

4. A Point Minus a Point Changes Type

Check 4.1: with $P = (3, 1)$, $Q = (1, 4)$ and $R = (-2, 2)$, the gap between $(P - Q) + (Q - R)$ and $P - R$ was **0**. Differences of points chain like arrows.

Check 4.2 (the check that failed): The first version treated every result as a point. Moving the origin shifts a point's coordinates and leaves a vector's alone, so the two need different tests. Moving the origin by $(10, -7)$:

Expression	Weights sum	As a point?	As a vector?	Verdict
$P - Q$	0	no	yes	a vector
$(P + Q)/2$	1	yes	no	a point
$2P - Q$	1	yes	no	a point
$3P - 2Q$	1	yes	no	a point
$P + Q$	2	no	no	meaningless
$2P + Q$	3	no	no	meaningless

Observation 4.3

Weights summing to 1 give a point, to 0 a vector, and anything else moves with the origin and means nothing. Here the minus does what the plus cannot — it takes two points and returns an object of another type. $P + Q$ is not unusual but undefined. Two points may be averaged and may not be added.

5. Infinity Minus Infinity Depends on the Road

Check 5.1: the difference of two quantities that both blow up, as $x \rightarrow 0$.

Expression	$x = 10^{-2}$	$x = 10^{-4}$	$x = 10^{-6}$
$1/x - 1/x$	0	0	0
$1/x - 1/\sin x$	-0.00166669	-1.66667×10^{-5}	-1.66707×10^{-7}
$1/x - 1/\tan x$	+0.00333336	$+3.33333 \times 10^{-5}$	$+3.33297 \times 10^{-7}$
$1/x^2 - 1/x$	+9900	$+9.999 \times 10^7$	$+9.99999 \times 10^{11}$

Observation 5.2

All four look like infinity minus infinity, and the answers are zero, zero from below, zero from above, and a divergence. The symbol does not distinguish them.

6. Subtracting a Divergence Depends on the Scheme

Check 6.1: cut off a divergent integral at L and remove the divergence three ways.

Scheme	Finite part left behind
Subtract $\ln L$	+0.0000000000
Subtract $\ln(2L)$	-0.6931471806
Subtract $\ln(L/2)$	+0.6931471806

Here $\ln 2 = 0.6931471806$.

Observation 6.2

All three remove the same divergence. And what is left over differs by $\ln 2$ — and no computation inside a scheme can say which is right. The same shape as Paper 227, where the symbol did not settle whether a bound is reached.

7. The Unary Minus Is Not Subtraction

Observation 7.1

Expression	Arguments	What it does
$a - b$	two	an operation: undo an addition
$-a$	one	a name: the element that gives 0
a^{-1}	one	the same idea, written for a product
$A \setminus B$	two	an operation, but it undoes nothing

Observation 7.2

$-a + a = 0$ is a definition and not a computation. One symbol holds a binary job and a unary job — and saying minus does not say which.

8. The Census and the Separator

Check 8.1:

Which minus	Undoes an addition?	Output type	Unique?
$a - b$ in the integers	yes	same	yes
$a - b$ in the counting numbers	only when $a \geq b$	same	yes, when defined
Point minus point	yes	a vector	yes
Set difference	no	same	yes
The unary $-a$	not an operation	same	yes
Infinity minus infinity	no such addition	anything	no (the path)
Renormalisation	removes a divergence	same	no (the scheme)

Observation 8.2 (the separator)

What separates the senses of minus is three questions — whether it truly undoes an addition, whether the output has the same type, and whether the answer is unique. The symbol answers none of them.

Observation 8.3 (the fence)

The fence around this can be subtracted is having settled what is supposed to come back. Outside it the sentence is not even false but empty — the same shape as this divides in Paper 229 and these can be added in Paper 228.

Observation 8.4

Paper 228 closed with before asking whether things can be added, ask in which variable. The answer here is its mirror — before subtracting, ask what is supposed to come back.

9. The Census and the Close

What this paper measured:

1. The counting numbers fail about half the time (Check 2.1): 50.0575%.

2. Set difference does not undo union (Check 3.2): **44.0115%, only when disjoint, 0 counterexamples.**
3. The first check was wrong (Check 4.2): **points and vectors need different tests.**
4. Three classes by the sum of the weights (Check 4.2): **1, 0, and neither.**
5. Four answers for infinity minus infinity (Check 5.1).
6. The schemes differ by $\ln 2$ (Check 6.1): **± 0.6931471806 .**
7. The unary $-a$ is not an operation (Observation 7.1): **it is a name.**

Close: We are taught that subtraction undoes addition. With sets it does not. Put A and B together and take B away, and A does not come back — what belonged to A all along is thrown out with the rest. It came back in 44.0115% of 200000 pairs, and only when the two were disjoint. And a point minus a point gives something that is not a point. Weights summing to one give a point, to zero a vector, and anything else moves with the origin and means nothing — two points may be averaged and may not be added. Infinity minus infinity depends on the road, and subtracting a divergence leaves a $\ln 2$ that the scheme decides. And $-a$ is not subtraction at all — it is a name, and $-a + a = 0$ is a definition, not a calculation. One symbol was holding the job of undoing, the job of changing type, and the job of naming. So before asking whether something can be subtracted, one must ask what is supposed to come back — and nothing guarantees that it is the same kind of thing that went in.

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