

Learn from a Book with the Proofs Removed and the Equals Sign Loses Its Type

— Carr's Synopsis and Ramanujan: Four Different Claims Written with One Equals Sign, and One Inequality No Computation Can Ever Refute — [Paper 232]

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— *From thinking to syncing.* —

ABSTRACT

The book Ramanujan learned from was some five thousand results with the proofs stripped out. When a proof goes, the fence goes with it — and when the fence goes, the symbol no longer carries its type. This paper sorts the equals signs he wrote into four kinds and checks each in numbers. **NO NEW MATHEMATICAL THEOREM AND NO NEW LAW IS CLAIMED.** First, some are exactly true. His series for $1/\pi$ buys about eight digits a term, the correct digits running $7 \rightarrow 15 \rightarrow 23 \rightarrow 31 \rightarrow 39 \rightarrow 47$ (Section 3). Second, some are only asymptotic. For the partitions the absolute error grows from 6.10 to 4.71×10^{43} while the relative error falls from 14.53% to 0.998% (Section 4). Third, this is the core. The partial sums of $1 + 2 + 3 + \dots$ reach 5,000,000,050,000,000 at 10^8 terms — what agrees with $-1/12$ is $\zeta(-1)$, to a difference of 0, and not the sum (Section 5). Fourth, some need a proof of convergence. The nested radical reaches 2.999999999986 at depth 40 — and nowhere does the symbol say that it converges (Section 6). Fifth, one inequality no computation can refute. $\pi(x) < \text{li}(x)$ holds at every x out to 10^7 , the gap reaching 339.41 — and Littlewood proved it fails infinitely often (Section 7).

Scope of this paper (scope note): NO NEW MATHEMATICAL THEOREM AND NO NEW LAW IS CLAIMED — the character of Carr's Synopsis, Ramanujan's series for $1/\pi$, the Hardy–Ramanujan asymptotic for the partitions, $\zeta(-1) = -1/12$ and Littlewood's theorem are all standard. No biography is offered — neither Ramanujan's life nor the bibliography of Carr's book is treated. No claim about teaching is made — whether one should learn from a book without proofs is not asked. No general theory of divergent series is offered — Ramanujan summation and Abel summation are cited, not developed. Littlewood's theorem is not proved — and no first crossing is given, because none is known. Nobody is being faulted — the claim here is about the symbol and not about the man. Relation to earlier papers: Paper 16 counted six distinct roots of the equals sign — this paper applies that census to one writer's own formulas. Paper 211 showed that adding more terms takes you further from the answer — Section 4 is an instance of it. Paper 227 showed that the symbol does not settle whether a wall can be touched — Section 7 is its most extreme case. Paper 223 showed that everything can pass every test and structure still be there — $\pi(x) < \text{li}(x)$ is its number-theoretic twin. Paper 31 treated Ramanujan's series from the side of what π is — this paper takes the side of what the equals sign is, and they do not overlap. What is added is measuring the digits gained per term, showing the absolute and relative errors of the partition formula moving in opposite directions, setting the partial sums beside $\zeta(-1)$, following the nested radical by depth, and measuring $\text{li}(x) - \pi(x)$ out to 10^7 .

1. Notation and Position

Paper 16 counted **six distinct roots** of the equals sign — a definition, a consequence of substitution, an identity, a conditional equation, an empirical law, and one that is dimensionally empty. This paper applies that census **to the formulas of one writer**. The question is **which equals sign is of which type**, not **who was right**.

Position: Paper 31 treated these series from the side of π — **this one takes the side of the equals sign**.

2. What Carr's Book Left Out

Observation 2.1

Carr's *Synopsis* sets out some five thousand results **with little or no proof**. It was Ramanujan's principal entry into the subject.

Observation 2.2

When a proof goes, something else goes with it — the fence. Where a result holds, whether a series converges, in what sense two things are equal: these live only inside the proof. Keep the results alone and the symbols remain without their types.

This is not a complaint. Carr's book was written as a revision summary, and omitting proofs is right for that purpose. The point is not about the book but about the symbol carrying no type.

3. An Equals Sign That Is Exactly True

Check 3.1: sum Ramanujan's series for $1/\pi$ term by term.

Terms	Error	Correct digits
1	7.64×10^{-8}	7
2	6.4×10^{-16}	15
3	5.68×10^{-24}	23
4	5.24×10^{-32}	31
5	4.94×10^{-40}	39
6	4.74×10^{-48}	47

Observation 3.2

Each term buys exactly eight digits. This is a genuine equality: the series converges and its limit is $1/\pi$ itself.

4. An Equals Sign That Is Only Asymptotic

Check 4.1: compare the leading Hardy–Ramanujan term with the exact $p(n)$.

n	Exact $p(n)$	Absolute error	Relative error
10	42	6.10	14.53%
100	190569292	8.71×10^6	4.57%
1000	2.41×10^{31}	3.41×10^{29}	1.42%
2000	4.72×10^{45}	4.71×10^{43}	0.998%

Observation 4.2

The absolute error grows without bound and the relative error falls. Written with an equals sign the statement is false; written with a tilde it is a theorem — one symbol decides the truth value. An instance of Paper 211.

5. The Core: An Equals Sign That Is a Convention

Check 5.1: the partial sums of $1 + 2 + 3 + \dots$.

Terms added	Partial sum
10	55
10^4	50,005,000
10^6	500,000,500,000
10^8	5,000,000,050,000,000

Meanwhile $\zeta(-1) = -0.08333333333333333333$ and $-1/12 = -0.08333333333333333333$, differing by 0.

Observation 5.2

The partial sums on the left approach nothing. What agrees with $-1/12$ is the value of an analytically continued function, not a sum — the two are both correct and not the same statement. One equals sign is doing two different jobs.

Observation 5.3

In the census of Paper 16 this is the convention. Writing $= -1/12$ is not a discovery but the consequence of choosing one summation method — and which method was chosen is not written in the symbol.

6. An Equals Sign That Needs a Proof of Convergence

Check 6.1: evaluate the nested radical at increasing depth.

Depth	Value
5	2.755053261329895984801018
10	2.989920360631050804024012
20	2.999987880599493711392443
40	2.9999999999986179938154983

Observation 6.2

The value is 3. And nowhere in the symbol is it written that the nesting converges — that must be shown separately. Ramanujan himself posed it as a problem and not as a proof.

7. An Inequality No Computation Can Ever Refute

Check 7.1: measure $\text{li}(x) - \pi(x)$.

x	$\pi(x)$	$\text{li}(x) - \pi(x)$
10^2	25	+5.126141584
10^4	1229	+17.1372159
10^6	78498	+129.5491595
10^7	664579	+339.4050486

Observation 7.2

It never once fails. And in 1914 Littlewood proved that the sign changes infinitely often — and the first crossing has still not been located.

Observation 7.3

It passes every test that can be run, and it is false. The number-theoretic twin of Paper 223, where a test sees only what it was built to see, and the most extreme case of Paper 227, where the symbol does not settle whether the wall is touched.

8. The Separator and the Fence

Observation 8.1 (the separator)

What separates one equals sign from another is which fence was left out.

Statement	The type of its “=”	The fence that is missing
The series for $1/\pi$	exact	none
The partition formula	asymptotic	“as $n \rightarrow \infty$, in relative error”
$1 + 2 + 3 + \cdots = -1/12$	a convention	“under which summation”
The nested radical	exact	“this nesting converges”
$\pi(x) < \text{li}(x)$	false	“checked over a finite range”

Observation 8.2 (the fence)

The fence around this formula is correct is saying which type its equals sign has. Outside it the sentence is not even false but empty — the same shape as Papers 228, 229 and 230.

Observation 8.3

A proof is not only a guarantee of the result. It is also the vessel that carries the fence — so a summary with the proofs removed keeps the results and discards their types. Ramanujan's astonishment began from that state, and what Hardy mostly added was the fences.

9. The Census and the Close

What this paper measured:

1. The series for $1/\pi$ is exact (Check 3.1): **eight digits a term.**
2. The partition formula is asymptotic (Check 4.1): **4.71×10^{43} and 0.998%.**
3. The $-1/12$ is a convention (Check 5.1): **partial sums at 5×10^{15} , $\zeta(-1)$ agreeing to 0.**
4. The nested radical needs convergence (Check 6.1): **2.999999999986 at depth 40.**
5. $\pi(x) < \text{li}(x)$ is unbroken to 10^7 and false (Check 7.1).
6. In all five, the symbol does not give the type (Observation 8.1).

Close: The book that came into Ramanujan's hands had the results and not the proofs. To have no proof is to have no fence. Where a thing holds, whether it converges, in what sense it is equal — all of that lives inside the proof. And so his notebooks came to hold four different kinds of claim written with one equals sign. The series for $1/\pi$ is exactly true and buys eight digits a term. The partition formula is only asymptotic, its absolute error growing to 10^{43} . In $1 + 2 + 3 + \dots = -1/12$ the left side approaches nothing and the right is the value of another function. The nested radical equals 3, and that has to be shown outside the symbol. And $\pi(x) < \text{li}(x)$ holds as far as anyone can compute, and is false. Not one of them announces its type. A proof, then, is both a warrant for a result and the vessel that carries its fence. A summary throws the vessel away and keeps the result — so whoever learns from a summary receives the results and not their types. Much of what Hardy later supplied was not new results but the fences that had fallen away.

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