

“Mass” Is Seven Things, One of Which Goes Negative and One of Which Depends on the Direction of the Push

— Invariant Mass Is Not Additive and Nothing Weighs the Reduced Mass: the Separator Is What the Resistance Is Measured Against — [Paper 240]

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— *From thinking to syncing.* —

ABSTRACT

This body of work uses the word "mass" in 75 papers, 540 times, and has never once counted it — Paper 165 counted three equivalence principles and that is all. This paper is that census, computed from definitions alone, with no physical constant quoted. NO NEW PHYSICAL LAW AND NO NEW MATHEMATICAL THEOREM IS CLAIMED. First, invariant mass is not additive. The mass of a system of two photons of energy E runs $0 \rightarrow 0.0174530709967479E \rightarrow 1.0E \rightarrow 1.41421356237E \rightarrow 2.0E$ with the angle alone — and each part has mass zero (Section 2). Second, inertial mass is not one number. A push along the motion meets $\gamma^3 m$ and a push across it meets γm — 12.074512309 against 2.29415733871 at $v = 0.9$, and **11188.7303863** against 22.3662720421 at $v = 0.999$, a ratio of 500.25 (Section 3). Third, nothing weighs the reduced mass. μ is always below both, being 0.999000999001 even for $m_1 = 1$ and $m_2 = 1000$ (Section 4). Fourth, and this is the core. Effective mass runs from positive through infinite to negative: across a one-dimensional band it goes $0.5 \rightarrow 28.6493442493 \rightarrow \text{divergent} \rightarrow -28.6493442493 \rightarrow -0.5$ — at the top of the band a push to the right accelerates it left (Section 5). Fifth, this returns to where the work began. The $m = 4\pi$ of Paper 02 becomes a mass only once k is placed — and Paper 02 wrote that itself, in its note on units (Section 6).

Scope of this paper (scope note): NO NEW PHYSICAL LAW AND NO NEW MATHEMATICAL THEOREM IS CLAIMED — the definition of invariant mass, longitudinal and transverse mass, reduced mass, the effective mass of a band, and the ADM and Bondi masses are all standard. No measured value is quoted: every number here is computed from a definition in units with $c = 1$, and no physical constant and no nuclear datum is used. The equivalence principle is not treated; the agreement of inertial and gravitational mass is left to Paper 165. The origin of mass is not discussed; neither the Higgs mechanism nor confinement is treated. No general relativity is computed; the ADM and Bondi masses of Section 7 are named and not calculated. Renormalisation is not discussed; the bare mass is named only. Paper 02 is not corrected: what Section 6 quotes is the note Paper 02 wrote for itself, and this paper only generalises it. Relation to earlier papers: Paper 02 showed that the total mass of $\rho = r^{-2}$ over the unit ball equals the solid angle 4π , and set that fence itself in a note on units; this paper puts the same question to every m in physics. Paper 165 counted three equivalence principles; this paper sets two of those masses beside the other five. Paper 196 counted four pressures and showed that sharing a unit guarantees nothing; this is its counterpart for mass. Paper 210 showed that four quantities go by the name of the speed of light; the shape is the same. What is added is the demonstration that a two-photon system's mass runs from 0 to $2E$ on angle alone, the six-row measurement of the longitudinal-to-transverse ratio, the tabulation showing the reduced mass always below both, the nine-point tracking of effective mass through positive, infinite and negative, and the dimensional check restoring k and R to the 4π of Paper 02.

1. Notation and Position

The question: when physics writes "mass m ", what resistance has been measured?

This body of work uses the word in 75 papers, 540 times, and has never counted it. Counting the named senses, relativistic mass, effective mass, longitudinal mass, bare mass, mass defect, ADM mass and reduced mass do not appear even once in the text.

Units: everything below takes $c = 1$ and is computed from definitions. No physical constant and no measured value is quoted — and without them, the seven still separate.

2. Invariant Mass Is Not Additive

Definition 2.1

the invariant mass of a system is given by $M^2 = E^2 - |\mathbf{p}|^2$.

Computation 2.2: for two photons of energy E at angle θ , $M^2 = 2E^2(1 - \cos \theta)$.

θ (degrees)	M/E	mass of each part
0	0	0
1	0.0174530709967479	0
60	1.0	0
90	1.4142135623731	0
180	2.0	0

Observation 2.3

two massless parts make a system with mass, and that mass is fixed by the angle alone — so mass is not the sum of the masses of the parts.

Observation 2.4

for the same reason a bound system has less mass than its parts — this paper quotes no nuclear data and therefore gives no value. What can be said is the consequence of the definition, and no more.

3. Inertial Mass Is Not One Number

Computation 3.1: apply a force F to a body moving at speed v . The acceleration is $F/(\gamma^3 m)$ along the motion and $F/(\gamma m)$ across it.

v	γ	along: γ^3	across: γ	ratio
0	1.0	1.0	1.0	1.0
0.5	1.154700538	1.53960071784	1.15470053838	1.333333333
0.9	2.294157339	12.074512309	2.29415733871	5.263157895
0.99	7.08881205	356.221711059	7.08881205008	50.25125628
0.999	22.36627204	11188.7303863	22.3662720421	500.2501251

Observation 3.2

the same body returns a different resistance depending on the direction of the push. At $v = 0.999$ the longitudinal resistance is 500 times the transverse — so "mass increases with speed" has the direction omitted.

Observation 3.3

"relativistic mass" fell out of use because it is not one number. Keeping $F = ma$ requires a different m for each direction — the modern convention fixes m as an invariant and puts γ on the momentum instead. This is a choice of convention and not a discovery.

4. Reduced Mass: Nothing Has It

Computation 4.1: the reduced mass of a two-body problem, $\mu = m_1 m_2 / (m_1 + m_2)$.

m_1	m_2	μ
1	1	0.5
1	10	0.909090909091
1	1000	0.999000999001
5	5	2.5

Observation 4.2

μ is always below both, and no body anywhere in the problem has that mass — μ is the mass of a fictitious body standing in for a relation. Here the word "mass" is not even a property of a thing.

5. The Core: Effective Mass Goes Negative

Computation 5.1: in a one-dimensional band $E(k) = -2t \cos(ka)$, measure the effective mass $m^* = \hbar^2 / (2ta^2 \cos(ka))$, with $\hbar = t = a = 1$.

ka (degrees)	m^*	sign
0	0.5	positive
80	2.87938524157	positive
89	28.6493442493	positive
90	divergent	the inflection of the band
91	-28.6493442493	negative
135	-0.707106781187	negative
180	-0.5	negative

Observation 5.2

one electron, in one band, carries positive, infinite and negative mass. At the top of the band a push to the right accelerates it left — the lattice is pushing back, and the electron has neither gained nor lost anything.

Observation 5.3

the "mass" here is not a property of a thing but the curvature of a dispersion relation. The definition $m^* = \hbar^2 / (d^2 E / dk^2)$ simply calls the reciprocal of a curvature by that name — standing in the same place as Paper 224 counting the roots of "curvature".

6. Mass as a Solid Angle: the Fence Paper 02 Set for Itself

Paper 02 showed that the total mass of $\rho = r^{-2}$ over the unit ball equals the solid angle 4π . And in the same paper it wrote:

$\rho = r^{-2}$ has dimensions $[L^{-2}]$, and to be a density it should be written $\rho = k/r^2$ with a constant k of dimension $[M/L]$. The statement $m = 4\pi$ holds under a choice of units, namely $k = 1$ and radius 1.

Computation 6.1: restore that k and the radius R : $m = 4\pi k R$.

k	R	$m = 4\pi kR$
1	1	12.566370614359
1	2	25.132741228718
3	1	37.699111843078
0.5	4	25.132741228718

Observation 6.2

4π is a solid angle. It becomes a mass once k is placed, and k is what carries $[M]$ — so the seventh root is a mass that borrows its dimension from a constant.

Observation 6.3

this is not a correction to Paper 02. Paper 02 set this fence itself — all this paper did was show that the same fence is needed by the other six. The discipline was already there at the first step.

Computation 6.4: and the module staircase of Papers 01 and 02, $m(n) = 4\pi/(3 - n)$.

n	$m(n)$	note
0	4.1887902047864	
1	6.2831853071796	
2	12.566370614359	equals the solid angle
2.5	25.132741228718	
2.9	125.66370614359	

$n = 2$ is the one exponent making the radial integral exactly 1. (QED)

7. Named Only

The following are not computed here. They are named, and given an address.

1. **Gravitational mass** (passive and active) — its agreement with inertial mass is left to Paper 165.
2. **Bare and renormalised mass** — only the latter is observed, and the former depends on how the regularisation is taken. Renormalisation is not discussed here.
3. **ADM and Bondi mass** — the first is measured over an initial surface, the second after the radiation has left, and the two do not agree in general. And the gravitational field has no local energy density — so "how much mass is here" can fail to be writable in principle. Not one line is computed here.

8. Separator and Fence

Observation 8.1 (the census)

name	resistance to what	the fact that bites
invariant mass	an invariant of energy and momentum	not additive
inertial mass	reluctance to accelerate under a force	varies with direction
gravitational mass	coupling to gravity	Paper 165
reduced mass	the relative motion of two bodies	nothing has it
effective mass	the curvature of a dispersion relation	goes negative
bare and renormalised	how the regularisation is taken	only the latter is seen
mass as a solid angle	the constant k that was placed	the note in Paper 02

Observation 8.2 (the separator)

what divides the seven is what the resistance is measured against. The symbol m names none of them — the same shape as f' in Paper 233, $d(x, y)$ in Paper 234, $-u''$ in Paper 235, " $= 0$ " in Paper 236 and $\sum a_n$ in Paper 239.

Observation 8.3 (the fence)

there are three. First, whether it can be added: invariant mass cannot (Section 2). Second, whether it is one number: inertial mass varies with direction (Section 3). Third, where its $[M]$ comes from: the solid angle borrows it from k (Section 6). None of the three is written in the symbol m .

Observation 8.4

Paper 196 wrote that sharing a unit guarantees nothing. For mass it is stronger still — the quantity can go negative, and it can have no owner at all.

9. Census and Closing

What this paper measured.

1. A two-photon mass running from 0 to $2E$ on angle alone (2.2).
2. A longitudinal-to-transverse ratio of 500.25 (3.1): at $v = 0.999$.
3. μ always below both (4.1).
4. Effective mass running $0.5 \rightarrow \text{divergent} \rightarrow -0.5$ (5.1).
5. $m = 4\pi kR$, with $[M]$ coming from k (6.1).
6. $m(n) = 4\pi/(3 - n)$, equal to the solid angle at $n = 2$ (6.4).

Closing: this body of work used the word "mass" in 75 papers, 540 times, and never once counted it. Counted, there are seven: one that does not add, one that changes with the direction of the push, one that goes negative, and one that nothing in the problem possesses. What divides them is what the resistance is measured against, and the symbol m does not write it. And the last of the seven is the starting point of this work itself. The $m = 4\pi$ of Paper 02 is a solid angle, and it becomes a mass once k is placed — and Paper 02 wrote that itself, in its note on units. This paper did not correct that note. It showed only that the same note is needed by the other six. The discipline was already there at the first step — this census merely caught up with it, two hundred and thirty-eight papers later.

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