

Before Asking Whether It Can Be Made into Addition, Ask Whether There Is a Binary Operation at All

— The Same Two Parts Give Different Wholes: Invariant Mass, Mixed Temperature and Quantum Entropy,
While Velocity Does Not — [Paper 241]

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— *From thinking to syncing.* —

ABSTRACT

Paper 228 asked whether a given "+" becomes real addition after a change of coordinate, and found exactly one case that cannot be repaired — non-commutativity. This paper asks the question before that: is the value of the whole a function of the values of the parts at all? Where it is not, there is no binary operation to repair. **NO NEW MATHEMATICAL THEOREM AND NO NEW LAW IS CLAIMED.** First, velocity is a function of its parts. At $v_1 = v_2 = 0.9$ the composition gives 0.994475138121547 , and the sum of rapidities gives the same value — which is why Paper 228 could repair it by changing coordinates (Section 2). Second, mass is not. For a system of two photons the parts have mass 0 and 0 in every row, while the whole runs $0 \rightarrow 1.0E \rightarrow 1.41421356237E \rightarrow 2.0E$ (Section 3). Third, temperature is not either. Mixing 300 K with 400 K gives $350 \rightarrow 375 \rightarrow 325 \rightarrow 399$ K while the two temperatures stay the same (Section 4). Fourth, and this is the core. For quantum entropy the parts constrain the whole not at all: with $S(A) = S(B) = 1.000000000$ bit throughout, $S(AB)$ is **0.000000000** or **2.000000000** — the two ends of the range available to two qubits (Section 5). Fifth, of the three clocks of Paper 118 only one falls. Coordinate time is a function of its parts, and only dS falls, and only in the quantum case (Section 6).

Scope of this paper (scope note): NO NEW MATHEMATICAL THEOREM AND NO NEW LAW IS CLAIMED — relativistic velocity composition, the definition of invariant mass, the distinction between extensive and intensive quantities, and the subadditivity of von Neumann entropy on entangled systems are all standard. No measured value is quoted; every number is computed from a definition. **No theory of entanglement is built**; Section 5 sets two states side by side and treats neither measures nor classification of entanglement. **No thermodynamics is discussed**; Section 4 uses only the definition of a heat-capacity-weighted mean. **The non-commuting case is not treated**; that is the core of Paper 228, and this paper **does not enter the lower level**. **No interpretation is placed on "the parts do not fix the whole"**; neither holism nor emergence is claimed, and what is shown is the formal fact that a map is not a function, nothing more. **Relation to earlier papers**: Paper 228 asked whether a change of coordinate turns a "+" into addition and named non-commutativity as the only irreparable case; this paper asks one level above and places 228 in the lower level. Paper 162 showed that the whole can be less known than its parts; Section 5 reads that same fact as a failure of the map from the parts. Paper 240 showed that invariant mass is not additive; this paper answers why no coordinate can repair it. Paper 118 separated dt , $d\tau$ and dS as three logical types; Section 6 says which of them falls. What is added is the explicit statement that the question has two levels, the placing of velocity, mass, temperature and entropy against one criterion, the numerical demonstration that an entangled pair and an independent pair have **identical marginal entropies**, and the check that only one of the three clocks falls.

1. Notation and Position

Setting: let A and B be systems and AB their composite. For a quantity Q , ask **two questions in order**.

1. **Level one**: is $Q(AB)$ a function of $(Q(A), Q(B))$?
2. **Level two** (only if level one is yes): can a change of coordinate turn that function into **addition**?

Paper 228 worked entirely at level two. It counted eight plus signs, found that seven become real addition after a change of coordinate, and that exactly one cannot be repaired, namely non-commutativity.

But some quantities fall at level one. There no binary operation exists to be repaired — it is not even a broken addition. This paper is that census.

No interpretation is offered: "the parts do not fix the whole" is a formal fact, and neither holism nor emergence is drawn from it here.

2. The Contrast: Velocity Is a Function of Its Parts

Computation 2.1: compare the relativistic composition $v_1 \oplus v_2 = (v_1 + v_2)/(1 + v_1 v_2)$ with the sum of rapidities, in units with $c = 1$.

v_1	v_2	$v_1 \oplus v_2$	$\tanh(\operatorname{artanh} v_1 + \operatorname{artanh} v_2)$
0.5	0.5	0.8	0.8
0.9	0.9	0.994475138121547	0.994475138121547
0.99	0.5	0.996655518394649	0.996655518394649

Observation 2.2

given the two parts, the whole is determined, so a binary operation exists — which is why Paper 228 could move it to a coordinate where it adds. The phrase "the addition of velocities is broken" is wrong: what was broken was the coordinate.

3. Mass: the Same Parts, Different Wholes

Computation 3.1: measure the invariant mass of a system of two photons of energy E at angle θ .

θ (degrees)	mass of part 1	mass of part 2	mass of the whole
0	0	0	0
60	0	0	$1.0E$
90	0	0	$1.41421356237E$
180	0	0	$2.0E$

Observation 3.2

the parts are exactly the same in every row and the whole is not — so $(m_1, m_2) \mapsto M$ is not a function.

Observation 3.3

this answers Paper 240. That paper showed invariant mass is not additive; what this one adds is why no coordinate repairs it. There is nothing to repair: for any transformation f , $f(M)$ cannot be determined by $f(m_1)$ and $f(m_2)$, because M is not a function of m_1 and m_2 .

4. Temperature: the Same Two Temperatures, Different Results

Computation 4.1: mix bodies of heat capacity C_1 and C_2 at 300 K and 400 K, giving $T = (C_1T_1 + C_2T_2)/(C_1 + C_2)$.

T_1	T_2	C_1	C_2	after mixing
300	400	1	1	350
300	400	1	3	375
300	400	3	1	325
300	400	1	99	399

Observation 4.2

the two temperatures are identical in every row and the result runs from 325 to 399 — temperature is intensive, and the temperatures of the parts do not fix the temperature of the whole. What fixes it is heat capacity, which is not a temperature.

5. The Core: the Parts Constrain the Whole Not at All

Computation 5.1: take two states of two qubits and measure the von Neumann entropy in bits.

state	$S(A)$	$S(B)$	$S(AB)$
Bell (pure, maximally entangled)	1.000000000	1.000000000	0.000000000
two independent maximally mixed	1.000000000	1.000000000	2.000000000

Observation 5.2

the parts agree to the bit, and the whole is 0 or 2 — and 0 and 2 are the two ends of the range $S(AB)$ can occupy for two qubits. So the parts do not merely fail to determine the whole; they constrain it not at all.

Observation 5.3

Paper 162 wrote that the whole can be less known than its parts. This is the same fact read from the side of the map — one fact, two places to stand.

Observation 5.4

no holism and no emergence is drawn from this. What is shown is the formal fact that $(S(A), S(B)) \mapsto S(AB)$ is not a function — any interpretation lies outside that fact.

6. Of the Three Clocks, Only One Falls

Paper 118 separated dt , $d\tau$ and dS as three distinct logical types. Apply the level-one question.

clock	a function of the parts?	note
coordinate time dt	yes	a difference of coordinates, and it adds
proper time $d\tau$	yes, once a path is given	but the endpoints do not fix it (Paper 234)
entropy dS	classically yes	quantum mechanically no (Section 5)

Observation 6.1

of the three, only dS falls at level one, and only in the quantum case. Proper time fails for a different reason, namely that the endpoints do not determine the path — the reversed triangle inequality of Paper 234 was that failure showing itself.

Observation 6.2

so the three clocks differ for three different reasons. One adds plainly, one depends on the path, and one is not constrained by its parts.

7. Separator and Fence

Observation 7.1 (the two-level census)

quantity	level one: a function of the parts?	level two: coordinates into +?
velocity	yes	yes (rapidity, Paper 228)
parallel resistance	yes	yes (conductance, Paper 228)
non-commuting operators	yes	no (the core of Paper 228)
invariant mass	no	falls before the question
temperature	no	the same
quantum entropy	no	the same

Observation 7.2 (the separator)

what divides level one is whether the whole is fixed by the values of the parts. When something else fixes it — an angle, a heat capacity, an entanglement — that something lies outside the map — and the notation does not write what lies outside.

Observation 7.3 (the fence)

the sentence " Q is not additive" carries two different meanings. One is there is an operation and it is not $+$, as for velocity and parallel resistance, which a coordinate repairs. The other is there is no operation, as for mass, temperature and quantum entropy, which nothing repairs. Confusing the two means calling a repairable problem unrepairable, and trying to repair an unrepairable one.

Observation 7.4

Paper 196 wrote that sharing a unit guarantees nothing. This stands beside it — having two of the same quantity guarantees nothing either.

8. Census and Closing

What this paper measured.

1. Velocity as a function of its parts (2.1): composition and rapidity sum agreeing at 0.994475138121547.
2. Mass not so (3.1): parts 0 and 0 throughout, whole from 0 to $2.0E$.
3. Temperature not so (4.1): 300 and 400 throughout, result from 325 to 399.
4. Entropy: the parts constraining nothing (5.1): $S(A) = S(B) = 1$ throughout, $S(AB)$ at 0 and 2.
5. Only dS falling at level one among the three clocks (6.1).

Closing: Paper 228 asked whether a "+" becomes real addition after a change of coordinate and found exactly one irreparable case. What this paper shows is that some quantities fall before that question is reached. Two photons, both of mass zero, move the mass of the whole from 0 to $2E$. 300 K and 400 K, unchanged, move the mixed result from 325 to 399. And one bit and one bit put the whole at 0 or at 2 — the two ends of what is available. These are not broken additions. There is nothing there to make into an addition. When writing " Q is not additive", which of the two must be said — one is repaired by a coordinate and the other is not. And no interpretation is offered: what was shown is the formal fact that a map is not a function, and holism and emergence lie outside that fact.

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