

Do Not Divide by Zero Is Not One Prohibition

— Inverses, Zero Divisors, the Projective Line, Indeterminate Forms, Conditioning and Floating Point: and
There Is Exactly One Ring in Which $2 = 1$ Is Sound — [Paper 242]

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September 2026 (Second Edition)

— *From thinking to syncing.* —

Abstract

Paper 229 counted five obstructions that keep division from undoing multiplication. This paper opens the word that sits one step earlier: is the prohibition do not divide by zero a single prohibition? No new mathematical theorem and no new law is claimed. First, the fake proof of $2 = 1$ divides by zero twice. The middle equation $(a + b)(a - b) = b(a - b)$ is an identity whose two sides differ by 0 across 200000 random draws. It carries no information (Section 2). Second, cancellation can fail at a nonzero element. In $\mathbb{Z}/6$ we have $2 \cdot 1 = 2 \cdot 4$ while $1 \neq 4$, and the triples violating cancellation number **24** in $\mathbb{Z}/6$ and **204** in $\mathbb{Z}/12$: in $\mathbb{Z}/100$, **59.60%** of the nonzero elements cannot be cancelled (Section 3). Third, on the projective line $1/0$ is defined. Approaching along 12 directions at radius 10^{-6} , the spread of $|1/z|$ is **3.49×10^{-10}** . What fails is $0/0$ (Section 4). Fourth, in analysis $0/0$ depends on the path. The same shape returns 1, 0.5, $+\infty$ and 0 (Section 5). Fifth, this is the core of the paper. Two neighbourhoods of one and the same great circle return different answers. The constant-width band gives **0.3364** and the wedge gives **0.5048**: the limits separate into $1/3$ and $1/2$ (Section 6). Sixth, in floating point the division is total. $+0.0 = -0.0$ holds while the reciprocals separate into $+\infty$ and $-\infty$ (Section 7). Seventh, there is exactly one ring in which $2 = 1$ is sound: the zero ring (Section 8).

Scope of this paper (scope note) : No new mathematical theorem and no new law is claimed: that 0 is not invertible in a field, that cancellation fails in a ring with zero divisors, that $1/0 = \infty$ is defined on the Riemann sphere, the Borel--Kolmogorov paradox, and the signed zero of IEEE 754 are all standard. No measured value is cited: every number here is computed from a definition or counted with a pseudorandom sample. No total division algebra is constructed: Section 7 only reads the behaviour of an existing standard. No general theory of conditioning is built: Section 6 places two neighbourhoods side by side and does not treat regular conditional probability. No collection of fallacies is assembled: exactly one fake proof is dissected. No meaning is attached to the existence of systems in which one may divide by zero. It is not claimed that any one of them is the true definition, nor that the school rule is wrong. What is shown is the formal fact that the reason for the prohibition differs from place to place. Relation to earlier papers: Paper 229 counted five obstructions to division. This paper opens one of them and finds six inside. Paper 16 showed that the equals sign has six roots: Section 2 rereads the fake proof as treating an identity as a conditional equation. Paper 236 showed that the same $= 0$ carries five kinds of solution set: Section 8 turns the same question toward the conclusion. Paper 164 showed that probability has four addresses: Section 6 measures the non-uniqueness of conditioning on a null event. Paper 154 showed that a limit written without an order is not a quantity: Section 6 is one instance. What is added is the separation of the prohibition into six roots, the numerical demonstration that the licence is injectivity rather than being nonzero, the count showing that two neighbourhoods of one great circle return different limits, and the confirmation that exactly one ring makes $2 = 1$ sound.

1. Notation and placement

Setting: read the quotient b/a as the x solving $ax = b$. Under that reading, **cannot divide by zero** is one of the following.

1. **No solution exists** (there is no inverse).
2. **The solution is not unique** (cancellation fails, or the family of approximations has not been specified).
3. **A value is determined, but the limit depends on the path.**
4. **There is no prohibition at all** (the operation is total).

The rule taught at school names only the first of these. This paper counts the rest.

The merits of the rule are not argued: only where each thing breaks is counted.

2. A field: no inverse, and the fake proof divides twice

Set out the familiar fake proof. Assume $a = b$; multiply by a to get $a^2 = ab$; subtract b^2 to get $a^2 - b^2 = ab - b^2$; factor to get $(a + b)(a - b) = b(a - b)$; divide by $(a - b)$ to get $a + b = b$, that is $2b = b$; divide by b to get $2 = 1$.

Check 2.1: with $a = b$, measure both sides of the middle equation over 200000 random draws.

quantity	value
$\max (a+b)(a-b) - b(a-b) $	0.000×10^0
$\max (a+b)(a-b) $	0.000×10^0
draws out of 200000 satisfying $2b = b$	0

Observation 2.2

The middle equation is an identity, and both sides are identically 0. What the fake proof does is extract information from an equation that has none. In the classification of Paper 16 it treats an identity as a conditional equation. What was broken is the standing of the equals sign, before any rule about division.

Observation 2.3

And $2b = b$ forces $b = 0$. The final step, dividing both sides by b , divides by zero as well: the proof divides by zero twice, and only the first is ever named.

3. Zero divisors: cancellation fails at a nonzero element

Check 3.1: in $\mathbb{Z}/n$, count the nonzero elements usable for cancellation, that is those for which $x \mapsto cx$ is injective.

n	nonzero	cancellable	not cancellable	share
5	4	4	0	0.0000
6	5	2	3	0.6000
12	11	4	7	0.6364
30	29	8	21	0.7241
100	99	40	59	0.5960

Check 3.2: count all ordered triples with $c \neq 0$, $x \neq y$ and $cx = cy$. There are **24** in $\mathbb{Z}/6$ and **204** in $\mathbb{Z}/12$. The smallest instance is $2 \cdot 1 = 2$ and $2 \cdot 4 = 2$ with $1 \neq 4$.

Observation 3.3

The licence for cancellation is not that $c \neq 0$. It is that $x \mapsto cx$ is injective. In a field the two coincide; in a general ring they do not.

Observation 3.4

Hence the rule that one may divide by anything nonzero is correct only in a field. In $\mathbb{Z}/100$, **59.60%** of the nonzero elements are unusable: a majority.

Observation 3.5

Paper 229 showed that in $\mathbb{Z}/12$ the equation $6x = 1$ has no solution while $6x = 0$ has six. This paper looks at the same ring from the side of cancellation: same ring, different question.

4. The projective line: $1/0$ is defined

Check 4.1: take $z = \varepsilon e^{i\theta}$ along 12 directions and measure $|1/z|$.

ε	$ 1/z $ min	$ 1/z $ max	spread
10^{-2}	1.000000×10^2	1.000000×10^2	2.842×10^{-14}
10^{-4}	1.000000×10^4	1.000000×10^4	3.638×10^{-12}
10^{-6}	1.000000×10^6	1.000000×10^6	3.492×10^{-10}

Observation 4.2

The manner of divergence does not depend on the direction. One point at infinity therefore suffices to define $1/0 = \infty$: on the Riemann sphere, dividing by zero is not forbidden.

Check 4.3: in the same limit, measure $0/0$ in the form $(az)/(bz)$.

a	b	limit	spread over directions
1	1	1.000000000000	1.11×10^{-16}
2	1	2.000000000000	2.22×10^{-16}
1	4	0.250000000000	2.78×10^{-17}
-3	7	-0.428571428571	1.11×10^{-16}

Observation 4.4

What cannot be defined on the projective line is not $1/0$ but $0/0$ and $\infty - \infty$. The prohibition did not vanish. Its address moved.

5. Analysis: $0/0$ depends on the path

Check 5.1: measure five expressions that take the shape $0/0$ as $x \rightarrow 0$.

expression	$x = 10^{-2}$	$x = 10^{-3}$	$x = 10^{-4}$	limit
$\sin x/x$	0.999983333	0.999999833	0.999999998	1
$(1 - \cos x)/x^2$	0.499995833	0.499999958	0.499999997	0.5
x/x^3	1.0×10^4	1.0×10^6	1.0×10^8	$+\infty$
$e^{-1/x^2}/x$	0.000000000	0.000000000	0.000000000	0
$(e^x - 1)/x$	1.005016708	1.000500167	1.000050002	1

Observation 5.2

All five carry the shape $0/0$. The values separate into 1, 0.5, $+\infty$ and 0. What separates them is not the symbol but which function approaches zero.

Observation 5.3

The prohibition here has a different root from the algebraic one. In algebra no inverse exists; in analysis the limit is not unique: a question of existence against a question of uniqueness.

6. The core: two neighbourhoods of one great circle differ

Setting: take the uniform distribution on the unit sphere and condition on **one great circle**, the intersection with the plane $y = 0$. That circle has measure zero, so the conditioning can only be defined as a limit of neighbourhoods.

Check 6.1: draw $N = 4000000$ points and measure the distribution of latitude in two neighbourhoods of the same circle.

neighbourhood	sample	share with latitude below 30°	limit
constant-width band $ y < 0.01$	40024	0.3364	uniform, $1/3 = 0.3333$
wedge, longitude below 0.01	25532	0.5048	cos weighted, $\sin 30^\circ = 0.5000$

Observation 6.2

The great circle is one and the same. Only the neighbourhood differs: and the answers separate into $1/3$ and $1/2$.

Observation 6.3

Conditioning on a null event is therefore not undefined. It is not unique. Yet another root, distinct from the previous four. A sentence saying we conditioned on the great circle has specified nothing.

Observation 6.4

Paper 154 wrote that a limit with no order written down is not a quantity. Here what is missing is not an order but the family of approximations: the same shape of fence, standing in a different place.

7. Floating point: division is total, equality is not identity

Check 7.1: measure the behaviour of IEEE 754 double precision.

expression	result
$+0.0 = -0.0$	true
$1/(+0.0)$	$+\infty$
$1/(-0.0)$	$-\infty$
$0.0/0.0$	NaN
$\infty - \infty$	NaN
$\text{NaN} = \text{NaN}$	false

Observation 7.2

Two values are equal under the equals sign, yet their reciprocals differ. Here the equals sign does not guarantee identity: a seventh use, not among the six roots of Paper 16.

Observation 7.3

In IEEE 754 division by zero is defined everywhere. There is no prohibition; $\pm\infty$ and NaN are returned instead: but NaN is not equal to itself. That is the price of totality.

8. There is exactly one ring in which $2 = 1$

Check 8.1: over n from 1 to 2000, count those for which $2 = 1$ holds in $\mathbb{Z}/n$. Exactly one qualifies, $n = 1$.

Observation 8.2

$2 = 1$ forces $1 = 0$, so the whole ring collapses to $\{0\}$. That is the zero ring. There 0 is invertible, since $0 \cdot 0 = 0 = 1$, and division by zero is defined.

Observation 8.3

The fake proof is therefore sound inside the zero ring. What is wrong is not the shape of the inference but the failure to say where it lives. Even the sentence that $2 = 1$ is false is undetermined until a place is named.

Observation 8.4

Paper 236 showed that the solution set of $x^2 = -1$ changes with the place it is asked to live in. This paper turns the same question toward the conclusion instead.

9. Separator and fences

Separator: what is being required.

place	requirement	can one divide by zero
field	existence of an inverse	no
ring with zero divisors	cancellation, that is injectivity	not even at a nonzero element
projective line	divergence to one point	yes, but not $0/0$
analysis	uniqueness of the limit	depends on the path
probability	a specified family of approximations	not uniquely
IEEE 754	totality	yes, as $\pm\infty$ and NaN

Fences:

1. It is not claimed that the six carry equal weight.
2. It is not claimed that any one of them is the true definition.
3. It is not claimed that the school rule is wrong: inside a field it is correct. What is missing is the address.
4. The merits of totalising the operation are not argued.
5. No collection of fallacies is assembled: one was dissected.

10. Roll call and closing

1. Information was drawn from an identity (Check 2.1): both the difference and each side are 0.
2. Division by zero happens twice (Observation 2.3): $2b = b$ forces $b = 0$.
3. Cancellation fails at nonzero elements (Checks 3.1 and 3.2): 59.60% in $\mathbb{Z}/100$, with 24 counterexamples in $\mathbb{Z}/6$.
4. On the projective line one may divide (Check 4.1): spread 3.49×10^{-10} .
5. In analysis the path decides (Check 5.1): 1, 0.5, $+\infty$, 0.
6. Two neighbourhoods of one circle differ (Check 6.1): 0.3364 against 0.5048.
7. In floating point the operation is total (Check 7.1): equality does not guarantee identity.
8. Exactly one ring makes $2 = 1$ sound (Check 8.1): the zero ring.

Closing: do not divide by zero was not a single prohibition. In a field there is no inverse; in a ring with zero divisors even a nonzero element may fail; on the projective line the division succeeds and $0/0$ fails; in analysis the path decides; in probability the answer is not unique; and in floating point there is no prohibition at all. And there is exactly one place where the fake proof is sound: the zero ring. The error lay not in the shape of the inference but in the failure to say where it was taking place. The school rule is correct. It simply carries no address.

Revision Record

Second edition (September 2026): no claim, theorem or check has been altered from the first edition. In the English references, the lines citing earlier papers of this corpus still carried their Japanese titles; they now carry the English titles. Nothing of substance has changed, and nothing has been withdrawn.

The dashes used as separators in the text were replaced by punctuation (October 2026). No claim, number or reference was changed.

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