

Two Limits Have No Answer Until the Order Is Written

— Spontaneous Symmetry Breaking Is a Question of Which of Volume and Field Is Sent First: the Same

Magnet Returns **0** and **0.957504024** — [Paper 245]

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— *From thinking to syncing.* —

Abstract

Paper 154 showed that a limit written without an order is not a quantity. This is its counterpart in physics: spontaneous symmetry breaking is a question of which of the volume and the field is sent to its limit first, and is therefore an order rather than a phenomenon. No new mathematical theorem and no new law is claimed. First, the smallest example. The function $x^2/(x^2 + y^2)$ returns **0** if x goes first and **1** if y does (Section 2). Second, a magnet does the same. Holding $h = 0$, the magnetisation is **0.000000000** at $N = 10$ and at $N = 10000$ alike (Section 3). Third, this is the core. Sending $N \rightarrow \infty$ first and only then $h \rightarrow 0$ gives **0.957504024**: the root of $m = \tanh(2m)$ (Section 4). Fourth, the crossover is visible in the numbers. Holding $h = 10^{-4}$, the magnetisation moves with N as **0.001730080** $\rightarrow$ **0.018263669** $\rightarrow$ **0.181087757** $\rightarrow$ **0.916808310** (Section 4). Fifth, this gives a number to the statement of Paper 175 that nothing breaks in a finite system. Nothing breaks not because the system lacks something, but because in a finite system $h \rightarrow 0$ acts first (Section 5). Sixth, the same accident occurs elsewhere: the classical limit, recurrence time, and refinement (Section 6).

Scope of this paper (scope note) : No new mathematical theorem and no new law is claimed: the order dependence of double limits, the self-consistency equation of the Curie--Weiss model, and the absence of phase transitions in finite systems are all standard. No measured value is cited: every number is computed exactly from the partition function. No theory of phase transitions is built: Sections 3 and 4 use one mean-field model and treat neither critical exponents nor the renormalisation group. Nothing is claimed about real magnets: mean field is an approximation and not a reality. No category theory is introduced: the theorem that filtered colimits commute with finite limits is not invoked, and only concrete examples are treated. No metaphysical answer is given to whether a symmetry is broken. What is shown is the formal fact that the answer is a function of the order. Relation to earlier papers: Paper 154 showed that a limit without an order is not a quantity. This paper gives numbers to its physical instance. Paper 175 wrote that nothing breaks in a finite system: Section 5 restates why, in the language of order. Paper 180 showed that the classical limit is four limits: Section 6 lines up the two of them that depend on order. Paper 81 wrote that the microscopic always returns but the universe ends first: Section 6 reads that as an order between volume and time. Paper 206 showed that spaces converge while dimension does not: another case of a limit failing to carry a property. What is added is the exact finite- N magnetisation with the crossover shown numerically, the restatement of nothing breaks as $\hbar \rightarrow 0$ acts first, and a list of limit pairs that depend on order.

1. Notation and placement

Setting: for a quantity $F(\varepsilon, \delta)$ with two parameters, consider **two limits**.

1. $\varepsilon \rightarrow 0$ first, then $\delta \rightarrow 0$
2. $\delta \rightarrow 0$ first, then $\varepsilon \rightarrow 0$

Nothing guarantees that the two agree. When they do not, the phrase in the limit as both go to zero has specified nothing.

No theory of phase transitions is built here. Cases where the order matters are counted.

2. The smallest example

Check 2.1: measure $F(x, y) = x^2/(x^2 + y^2)$ in both orders.

value held	$x \rightarrow 0$ first	$y \rightarrow 0$ first
10^{-2}	0.000000000	1.000000000
10^{-4}	0.000000000	1.000000000
10^{-6}	0.000000000	1.000000000

Observation 2.2

The answers are 0 and 1. Both are correct: the question was incomplete. This function has no value at the origin. It has a value per path.

3. A magnet: hold the field at zero and nothing happens

Setting: take N spins coupled equally to each other, the mean-field Curie--Weiss model, at inverse temperature $\beta = 2$, which lies **in the ordered phase**. Apply a field h and compute the magnetisation m **exactly from the partition function**.

Check 3.1: hold $h = 0$ and increase N .

N	magnetisation at $h = 0$
10	0.000000000
100	0.000000000
1000	0.000000000
10000	0.000000000

Observation 3.2

However large N becomes, the magnetisation is exactly 0. The reason is plain: at $h = 0$ up and down are perfectly symmetric and the sum cancels. For finite N this holds no matter how many spins there are.

4. The core: reverse the order and 0.957504024 appears

Check 4.1: hold h small but nonzero and increase N .

N	$h = 10^{-2}$	$h = 10^{-4}$	$h = 0$
10	0.171113372	0.001730080	0.000000000
100	0.916219247	0.018263669	0.000000000
1000	0.959229578	0.181087757	0.000000000
10000	0.959424500	0.916808310	0.000000000

Taking $N \rightarrow \infty$ first and then $h \rightarrow 0$, the magnetisation settles at the nonzero root of $m = \tanh(2m)$, which is **0.957504024**.

Observation 4.2

The same model, the same temperature, the same two limits. Only the order differs, and the answers separate into **0** and **0.957504024**.

Observation 4.3

Read a column of the table downward and the crossover appears. At $h = 10^{-4}$ the values run $0.0017 \rightarrow 0.018 \rightarrow 0.18 \rightarrow 0.92$. There is no threshold; the ratio of N and h decides.

Observation 4.4

Writing take the thermodynamic limit has specified nothing yet. Until the treatment of h is written down the answer is undetermined. What Paper 154 wrote on the mathematical side stands here unchanged.

5. Restating nothing breaks in a finite system

Paper 175 wrote that nothing breaks in a finite system. This paper puts numbers there and changes the wording.

Observation 5.1

It is not that nothing breaks. In a finite system $\hbar \rightarrow 0$ acts first. Saying nothing breaks makes it sound as though the finite system were missing something. It is not missing anything; its order is already fixed.

Observation 5.2

Why does a real magnet carry a magnetisation? Because N is finite but enormous and $\hbar$ is never exactly zero: the row with $\hbar = 10^{-2}$ and $N = 1000$ returns 0.959229578. It is not an ideal limit; a ratio decides it.

Observation 5.3

Only one mean-field model is used here. Neither critical exponents nor the renormalisation group are treated, and nothing is claimed about real magnets.

6. The same accident elsewhere

A list: pairs of limits where the order matters.

setting	the two limits	does order matter
spontaneous symmetry breaking	$N \rightarrow \infty$ and $\hbar \rightarrow 0$	**yes**
Poincare recurrence	$V \rightarrow \infty$ and $t \rightarrow \infty$	**yes**
classical limit	$\hbar \rightarrow 0$ and $t \rightarrow \infty$	**yes**
box dimension	$\epsilon \rightarrow 0$ and the thinness of the object	**yes**
uniform convergence	$n \rightarrow \infty$ and $x \rightarrow x_0$	**no, if uniform**

Observation 6.1

Only the last row differs. Assuming uniform convergence makes the order irrelevant, so order does not matter is a theorem and not a default. That is what Paper 197 was separating when it split convergence into four.

Observation 6.2

Paper 81 wrote that the microscopic always returns but the universe ends first. That too is an order: send $V \rightarrow \infty$ first and the recurrence time is infinite; send $t \rightarrow \infty$ first and the return is certain.

Observation 6.3

Paper 206 showed that a thin cylinder approaches a segment while its dimension stays at 2. That is a limit failing to carry a property, whereas this paper treats two limits at once: two different accidents.

7. Separator and fences

Separator: at what ratio the two limits are sent. Whether a combination such as Nh stays finite, goes to zero, or diverges. That is what decides the answer.

Fences:

1. Neither order is called correct: the question decides which one is needed.
2. It is not claimed that finite systems have phase transitions.
3. Nothing is claimed about real magnets: mean field is an approximation.
4. No metaphysical answer is given to whether a symmetry is broken.
5. No categorical theorem is invoked: only concrete examples.

8. Roll call and closing

1. The smallest example gives 0 and 1 (Check 2.1).
2. At $h = 0$ the magnetisation is 0 for every N (Check 3.1).
3. Reversing the order gives 0.957504024 (Section 4): the root of $m = \tanh(2m)$.

4. **The crossover is visible** (Check 4.1): at $\hbar = 10^{-4}$, 0.0017 rising to 0.92.

5. **Nothing breaks means $\hbar \rightarrow 0$ acts first** (Observation 5.1).

6. **Order stops mattering only under an assumption of uniformity** (Observation 6.1).

Closing: spontaneous symmetry breaking turned out to be an order rather than a phenomenon. In one model at one temperature, both 0 and 0.957504024 come out correctly. What separates them is not physics but the ratio at which N and $\hbar$ were sent. A sentence that says in the thermodynamic limit has specified nothing yet. And order does not matter is not a default but a theorem obtained by assuming uniformity.

Revision Record

Second edition (September 2026): no claim, theorem or check has been altered from the first edition. In the English references, the lines citing earlier papers of this corpus still carried their Japanese titles; **they now carry the English titles. Nothing of substance has changed, and nothing has been withdrawn.**

The dashes used as separators in the text were replaced by punctuation (October 2026). No claim, number or reference was changed.

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