

Splitting by Parity Is Not One Phenomenon

— Half-Integer Gamma, Time Reversal, the Support of a Green Function, the Sign of a Determinant: and One Search That Closes on the Even Side and Returns Nothing on the Odd — [Paper 247]

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September 2026 (Second Edition)

— *From thinking to syncing.* —

Abstract

Splitting by parity appears repeatedly in this body of work: the parity separation of ball volumes in Paper 115, and the different standing of even and odd orders in Paper 49. This paper asks whether they share a root: they do not. No new mathematical theorem and no new law is claimed. First, the two earlier splits are not the same. Paper 115 is split by the argument of the gamma function becoming a half-integer, and Paper 49 by the sign under time reversal (Section 2). Second, the gamma side shows in the power of π . The volume of the unit ball carries an integer power of π in even dimensions and a half-integer one in odd (Section 4). Third, only one side leaves a tail. In three dimensions the signal is exactly 0 for $t > 1.2$, while in two it still measures 6.53×10^{-8} at $t = 5.0$ (Section 5). Fourth, the sign of a determinant fixes orientability. $-I$ lies in $SO(n)$ for even n , and $\mathbb{RP}^n$ is orientable for odd n (Section 6). Fifth, this is the core. One and the same search closes on one side and returns nothing on the other. $\zeta(2k)/\pi^{2k}$ is found every time as the reciprocal of **6, 90, 945, 9450, 93555**, while $\zeta(3), \zeta(5), \zeta(7), \zeta(9)$ return nothing at all (Section 7). Sixth, perfect numbers have the same shape. Exhaustive search below 10^7 gives **6, 28, 496, 8128** and zero odd ones (Section 7). Seventh, the eight do not imply one another: the intuition that parity is merely a label mod 2 is false (Section 9).

Scope of this paper (scope note) : No new mathematical theorem and no new law is claimed: the parity of ball volumes, the validity of the Huygens principle in odd space dimensions, the orientability of $\mathbb{RP}^n$, Euler closed form for $\zeta(2k)$, the Euclid--Euler classification of even perfect numbers, and the centrality of the volume element in odd-dimensional Clifford algebras are all standard.No measured value is cited: every number is computed from a definition or obtained by exhaustive enumeration.Nothing is claimed about open problems: neither about a closed form for $\zeta(3)$ nor about the existence of an odd perfect number.What is shown is that one search closed on one side, and not that the other side is impossible.The result of a search is not found, not does not exist: the coefficients were bounded by 10^6 .No number theory is developed: Section 7 lists known values and applies one identical procedure to both sides.No meaning is attached to parity: neither side is called better, and it is not claimed that anything essential lives in $\text{mod } 2$.Relation to earlier papers: Paper 115 exhibited a parity separation in the recursion for ball volumes. This paper names its cause and shows it is a different root from the others.Paper 49 showed that even orders are conservative and odd ones dissipative. This paper writes that it and Paper 115 do not share a root.Paper 157 called the roll of the twos in this body of work. This paper counts what happens when that two is used as a modulus.Paper 166 showed that 4 and 8 share a root: the Clifford periodicity of Section 8 stands next to it.Paper 128 showed that solid angle and Euler characteristic agree only in three dimensions: Section 6 treats the parity of that χ .What is added is a count of eight parity separations, the naming of the cause of each, and a numerical demonstration that one identical search splits between the two sides.

1. Notation and placement

Setting: a quantity or a statement can change its character with the parity of an index n . This paper counts eight such cases and asks what separates each of them.

The question is not whether even and odd differ. It is whether splitting by parity is one thing.

No number theory is developed and nothing is claimed about open problems.

2. The two earlier splits do not share a root

Two parity separations already stand in this body of work.

paper	what splits by parity	the cause
Paper 115	the recursion for ball volume, the power of π	**the argument of $\mathbf{T}$ becoming a half-integer**
Paper 49	the standing of even and odd PN orders	**the sign under time reversal**

Observation 2.1

Neither implies the other. Time reversal symmetry does not follow from a half-integer gamma function, nor the converse: the same phrase, splitting by parity, covers two mechanisms.

Observation 2.2

This is the starting point. Reading them as one root because both are parity is exactly the accident this body of work has avoided. Paper 157 called the roll of the twos; this paper does the same for the side that uses two as a modulus.

3. Cancelling or surviving: symmetric integration

Check 3.1: put the cheapest parity first, integration over a symmetric interval.

<i>k</i>	$\int_{-1}^1 x^k dx$	parity
0	2	even
1	0	odd
2	2/3	even
3	0	odd
4	2/5	even
5	0	odd
6	2/7	even

Observation 3.2

The cause is plain: $(-1)^k$ appears as a sign and cancels in pairs. This is the shallowest of the eight roots, and the other seven are not explained by it. In practice it causes the most accidents: a symmetric arrangement produces a column of exact zeros, which is easily misread as convergence.

4. A half-integer argument of Γ : the volume of a ball

Check 4.1: list $V_n = \pi^{n/2}/\Gamma(n/2 + 1)$.

n	V_n	power of π
1	2.0	$\pi^{0.5}$
2	3.14159265359	π^1
3	4.18879020479	$\pi^{1.5}$
4	4.93480220054	π^2
5	5.26378901391	$\pi^{2.5}$
6	5.16771278005	π^3
7	4.72476597033	$\pi^{3.5}$
8	4.05871212642	π^4

Observation 4.2

Even dimensions give an integer power of π and odd ones a half-integer power. The separator is the argument of $\Gamma(n/2 + 1)$: an integer factorial for even n , and $\Gamma(1/2) = \sqrt{\pi}$ for odd. That square root shifts the power of π by a half.

Observation 4.3

This is a wholly different mechanism from the $(-1)^k$ of Section 3. No sign appears anywhere.

5. Tail or sharp: the Huygens principle

Setting: give an initial velocity supported within radius **0.2** and measure the signal at distance **1.0**, in two dimensions and in three.

t	two dimensions	three dimensions
0.9	1.472×10^{-7}	5.063×10^{-8}
1.0	7.430×10^{-7}	1.600×10^{-7}
1.1	8.078×10^{-7}	5.063×10^{-8}
1.2	5.070×10^{-7}	0
1.5	2.883×10^{-7}	0
2.0	1.851×10^{-7}	0
3.0	1.132×10^{-7}	0
5.0	6.533×10^{-8}	0

Observation 5.1

In three dimensions the signal is exactly 0 once $t > 1.2$. In two dimensions 6.53×10^{-8} still remains at $t = 5.0$: the wave has passed and it does not stop ringing.

Observation 5.2

The separator is the support of the Green function. In odd space dimensions the solution is a mean over a sphere and the support sits on the sphere. In even dimensions it becomes an integral over a ball and the support fills the interior. Another mechanism again, unlike the half-integer gamma and unlike $(-1)^k$.

Observation 5.3

Were space two-dimensional, speech would trail like ripples on a pond and conversation would not work. That is why Paper 64 listed sharp wave propagation among the criteria that select a dimension.

6. The sign of a determinant: is $-I$ a rotation

Check 6.1: ask whether the antipodal map $-I$ is a rotation in n dimensions. Here $\det(-I) = (-1)^n$.

n	$\det(-I)$	$-I \in SO(n)$	$\mathbb{RP}^n$ orientable	$\chi(S^n)$
1	-1	no	**yes**	0
2	+1	**yes**	no	2
3	-1	no	**yes**	0
4	+1	**yes**	no	2
5	-1	no	**yes**	0
6	+1	**yes**	no	2

Observation 6.2

Three separate facts follow from the one sign $(-1)^n$. Whether $-I$ is a rotation, whether projective space is orientable, and the Euler characteristic of the sphere. These three do share a root: and among the eight, only here is there a bundle.

Observation 6.3

Paper 128 showed that solid angle 4π and Euler characteristic $2\pi\chi$ agree only in three dimensions. This table is that χ alternating between 2 and 0.

7. The core: one search closes on one side and returns nothing on the other

Setting: ask whether $\zeta(s)/\pi^s$ is rational, applying exactly the same procedure to the even and the odd side. Search for an integer relation with coefficients bounded by 10^6 .

s	relation found	meaning
2	$(-6, 1)$	$\zeta(2)/\pi^2 = 1/6$
4	$(-90, 1)$	$1/90$
6	$(-945, 1)$	$1/945$
8	$(-9450, 1)$	$1/9450$
10	$(-93555, 1)$	$1/93555$
3	**none found**	no rational returned
5	**none found**	no rational returned
7	**none found**	no rational returned
9	**none found**	no rational returned

Observation 7.1

The search is identical, word for word. All five close instantly on the even side and all four return nothing on the odd. This is no proof that no rational exists on the odd side. It is a report that none was found with coefficients bounded by 10^6 .

Check 7.2: perfect numbers show the same shape. Enumerate exhaustively below 10^7 .

what was checked	result
perfect numbers below 10^7	6, 28, 496, 8128
of these, odd	0
the Euclid--Euler family, checked exactly	$\sigma(N) = 2N$ confirmed for $p = 2, 3, 5, 7, 13, 17, 19, 31, 61$
N at $p = 61$	2658455991569831744654692615953842176

Observation 7.3

Even perfect numbers are completely classified, in one-to-one correspondence with the Mersenne primes. Not one odd perfect number has been found: and whether any exists is unknown.

Observation 7.4

What the two share is that the definition makes no mention of parity. Neither the definition of $\zeta(s)$ nor that of a perfect number distinguishes even from odd. And still, only one side closed. The intuition that parity is merely a label mod 2 breaks here completely.

8. Does the algebra split: the Clifford volume element

Check 8.1: in the complex Clifford algebra on n generators, measure whether the volume element $\omega = e_1 e_2 \cdots e_n$ is central.

n	largest commutator with a generator	is ω central
1	0.000×10^0	**yes**
2	2.000	no
3	0.000×10^0	**yes**
4	2.000	no
5	0.000×10^0	**yes**
6	2.000	no

Observation 8.2

Since $\omega e_i = (-1)^{n-1} e_i \omega$, the volume element is central exactly for odd n : the centre becomes two-dimensional and the algebra splits in two.

Observation 8.3

Paper 231 showed that a law is lost at each step through the reals, complexes, quaternions and octonions, and Paper 166 that 4 and 8 share a root. Clifford periodicity stands beside both: mod 2 over the complexes and mod 8 over the reals.

9. Separator and fences

Separator: what is looking at the parity. Eight cases.

what looks at parity	how it shows	section
$(-1)^k$ standing as a sign	cancellation in symmetric integration	Section 3
a half-integer argument of Γ	the power of π shifts by a half	Section 4
the sign under time reversal	conservative and dissipative PN orders	Paper 49
the support of the Green function	a tail or a sharp front	Section 5
$\det(-I) = (-1)^n$	rotation, orientability, χ	Section 6
is the volume element central	does the algebra split	Section 8
is a closed form available	the two sides of ζ	Section 7
is the classification complete	the two sides of perfect numbers	Section 7

Observation 9.1

Of the eight, only the three in Section 6 imply one another. Not one of the rest follows from any other. Splitting by parity carries at least six distinct roots.

Fences:

1. Nothing is claimed about open problems: neither a closed form for $\zeta(3)$ nor the existence of an odd perfect number.
2. A search returns not found, never does not exist.

3. Neither parity is called better.
4. It is not claimed that anything essential lives in $\bmod 2$: only that the reason $\bmod 2$ bites differs from place to place.
5. Neither number theory nor representation theory is developed.

10. Roll call and closing

1. The two earlier splits differ (Section 2): half-integer gamma against time reversal.
2. Symmetric integration cancels through $(-1)^k$ (Check 3.1).
3. The power of π shifts by a half (Check 4.1): $\pi^{2.5}$ against π^3 .
4. Two dimensions leave a tail (Section 5): 6.53×10^{-8} at $t = 5.0$, against exactly 0.
5. Three facts from one sign (Check 6.1): rotation, orientability, χ .
6. One search closes on one side only (Section 7): 6, 90, 945, 9450, 93555 against four times none found.
7. Perfect numbers repeat the shape (Check 7.2): zero odd ones below 10^7 .
8. The volume element is central only for odd n (Check 8.1).

Closing: splitting by parity was not one phenomenon. A sign standing up, an argument of Γ becoming a half-integer, time reversal biting, the support of a Green function changing, the sign of a determinant deciding, an algebra splitting. And the strangest came last: a definition that never mentions parity, and yet one side is completely closed while no one has taken a single step into the other. The intuition that parity is merely a label $\bmod 2$ breaks there. The label is the same. What differs, from place to place, is why the label bites.

Revision Record

Second edition (September 2026): no claim, theorem or check has been altered from the first edition. In the English references, the lines citing earlier papers of this corpus still carried their Japanese titles; they now carry the English titles. Nothing of substance has changed, and nothing has been withdrawn.

The dashes used as separators in the text were replaced by punctuation (October 2026). No claim, number or reference was changed.

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