

Whose Cost Is the Demon's? Five People Put It in Five Places

— Measurement, Observation, Erasure, Room: Irreversible Gates Do Not All Lose the Same Amount, AND
Losing 1.188722 Bits and XOR Exactly 1.000000 — [Paper 248]

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— *From thinking to syncing.* —

Abstract

For more than a century Maxwell's demon has been asked where the cost lies. What this paper counts is not the answer but where the cost was placed: five people put it in five different places. No new mathematical theorem and no new law is claimed. First, one bit costs 2.870979×10^{-21} J at 300 K, that is 0.017919 eV. A 100 W device working at that bound could erase 3.4831×10^{22} bits per second (Section 3). Second, the Szilard engine returns exactly that much from one measurement. The isothermal work is $2.870978885079 \times 10^{-21}$ J, differing from $k_B T \ln 2$ by 6.5×10^{-35} J (Section 4). Third, this is the core. Irreversible gates do not all lose the same amount. AND, OR and NAND lose 1.188722 bits while XOR loses 1.000000 and erasure loses 1.000000 (Section 5). Fourth, gates that can be made reversible lose nothing. NOT, copying, Toffoli and Fredkin all lose 0.000000 bits, and the last two are permutations of 8 states that return to the identity when applied twice (Section 6). Fifth, the cost of a reversible AND is not erasure but room. Built from a Toffoli gate it loses 0 bits, and a third wire is required (Section 7). Sixth, the books balance. Over N cycles the work extracted and the cost of erasing the record cancel exactly, for every N (Section 8).

Scope of this paper (scope note) : No new mathematical theorem and no new law is claimed: Maxwell's thought experiment, the Szilard one-bit engine, Brillouin's cost of observation, Landauer's erasure bound, Bennett's resolution, and the Toffoli and Fredkin gates are all standard. No measured value is cited: every number is computed from a definition or obtained by exhaustive enumeration. The debate is not declared settled: generalisations of the second law that include measurement are still being studied after Bennett, and this paper only counts the structure of the debate. No theory of feedback control is built: Section 4 integrates the isothermal work and nothing more. It is not claimed that the second law is violated, nor that it is not. What is shown is the fact that the cost was placed in different places by different people. Nothing is said about real computing hardware: Sections 5 and 6 enumerate truth tables. Relation to earlier papers: Paper 131 split $k_B T \ln 2$ into three parts and wrote that an irreversible gate does not necessarily pay one bit. This paper puts a number on that (AND pays 1.188722). Paper 88 separated the coarse-graining entropy from the horizon entropy. What this paper treats is a third one, the entropy of a record. Paper 126 split the axiomatisation of Caratheodory into three stages. This paper views the same second law from the side of information. Paper 4 treated the resource bound on exhaustive search: Section 3 is where that bound comes from. Paper 143 showed that minimum entropy production is a fenced theorem rather than a principle. This paper likewise opens something that has been called a principle. What is added is a single table of where five people placed the cost, an exhaustive count of what irreversible gates lose, and a numerical demonstration that the cost of reversibility is room rather than erasure.

1. Notation and placement

Setting: divide a box of gas with a partition and place a small being that lets only the fast molecules through one way. A temperature difference appears at no apparent cost, and work can be drawn from it. The second law appears to be violated. Maxwell described this in a letter in 1867.

For over a century the question has been posed as: the cost must be somewhere. What this paper counts is not the answer but where the cost was placed.

The debate is not declared settled here, and it is not claimed either that the second law is violated or that it is not.

2. Five people, five different places

who	when	where the cost was said to lie
Maxwell	1867	**nowhere** (hence the paradox)
Szilard	1929	**measurement** , $k_B T \ln 2$ to learn one bit
Brillouin	1951	**the physical act of looking** , a photon is needed
Landauer	1961	**erasure** , $k_B T \ln 2$ to erase one bit
Bennett	1982	**erasure only** , measurement can in principle be free

Observation 2.1

All five are trying to save the second law. What differs is only which operation the cost was assigned to: measurement, the act of looking, or erasure.

Observation 2.2

Here is the separator. The phrase the demon's cost does not name an operation. Until one is named, no one is right or wrong; and once one is named, it is settled which of the five is being discussed.

Observation 2.3

Bennett's resolution worked because it separated logical from physical reversibility. Measurement can be made logically reversible; erasure cannot. The cost therefore falls on erasure and not measurement.

3. The price of one bit

Check 3.1: compute $k_B T \ln 2$ at 300 K.

quantity	value
$k_B T \ln 2$	$2.870979 \times 10^{-21} \text{ J}$
the same in electron volts	0.017919 eV
bits a 100 W device could erase at the bound	3.4831×10^{22} per second

Observation 3.2

The number is small; it is the size of thermal motion at room temperature. It is the value underneath the resource bound of Paper 4.

Observation 3.3

Paper 131 split this expression into three parts. k_B converts units, $\ln 2$ chooses the unit of information, and the physics is only in T . This paper stands on that and counts how many bits are paid.

4. The Szilard engine: work from one measurement

Setting: put one molecule in a box, insert a partition, **measure which side it is on**, and expand isothermally from that side. The work is the integral of pressure over volume.

Check 4.1: integrate the isothermal expansion from V to $2V$ over **2000001** points.

measured	value
isothermal work	$2.870978885079 \times 10^{-21} \text{ J}$
$k_B T \ln 2$	$2.870978885079 \times 10^{-21} \text{ J}$
difference	$6.5 \times 10^{-35} \text{ J}$

Observation 4.2

The work extracted equals the price of one bit exactly. Szilard read this as a cost attached to measurement. It balances what was extracted, so the reading is natural.

Observation 4.3

Yet the same number balances the cost of erasure as well. That two numbers agree does not decide which operation carries the cost: a balance alone fixes no address.

5. The core: irreversible gates do not all lose the same amount

Setting: assume uniform inputs and call the amount lost by a gate **the information of the input minus that of the output**. Enumerate the truth tables exhaustively.

gate	input	information of output	lost	injective
AND	2 bits	0.811278	1.188722	no
OR	2 bits	0.811278	1.188722	no
NAND	2 bits	0.811278	1.188722	no
XOR	2 bits	1.000000	1.000000	no
erasure	1 bit	0.000000	1.000000	no
NOT	1 bit	1.000000	0.000000	yes
copying	1 bit	1.000000	0.000000	yes

Observation 5.1

An irreversible gate pays one bit is not correct. AND loses 1.188722 bits while XOR loses exactly 1.000000: both are irreversible gates with two inputs and one output, and the amounts differ.

Observation 5.2

The reason is the bias of the output. The output of AND is zero three times and one once, carrying only 0.811278 bits. The output of XOR is balanced and carries exactly one bit. What is lost is fixed by the bias of the output, not of the input.

Observation 5.3

Paper 131 wrote that an irreversible gate does not necessarily pay one bit. This paper puts the number there. It pays 1.188722 bits, not one. And only erasure and XOR pay exactly one.

6. Gates that can be made reversible lose nothing

Check 6.1: examine the Toffoli and Fredkin gates on all 8 states.

gate	distinct outputs	lost	returns to identity twice
Toffoli	8/8	0.000000	**yes**
Fredkin	8/8	0.000000	**yes**

Observation 6.2

Both are permutations of eight states, and both are involutions. Applying either twice gives the identity, so no inverse needs to be built; the same gate serves.

Observation 6.3

Both are universal. Any logic circuit can be built from either alone: hence computation itself requires no erasure. That is the content of Bennett's conclusion.

7. A reversible AND: the cost is room, not erasure

Check 7.1: build AND from a Toffoli gate. Supply a third line set to zero and send $(a, b, 0)$ to $(a, b, a \wedge b)$.

input	output
(0, 0, 0)	(0, 0, 0)
(0, 1, 0)	(0, 1, 0)
(1, 0, 0)	(1, 0, 0)
(1, 1, 0)	(1, 1, 1)

Observation 7.2

Four inputs go to four different outputs; the map is injective. The information lost is **0.000000** bits: the same AND went from 1.188722 to zero by a change of construction alone.

Observation 7.3

It is not free. A third wire is required. The cost did not vanish; it changed form: from erasure into room. And room must eventually be cleared. The cost of erasure arrives then.

Observation 7.4

So there is a sixth place for the cost: when it is paid. Reversible computation did not remove the cost; it postponed it.

8. The demon's books

Check 8.1: over N cycles, place the work extracted beside the cost of erasing the record.

N	work extracted	cost of erasure	net
1	$2.870979 \times 10^{-21} \text{ J}$	$2.870979 \times 10^{-21} \text{ J}$	0
10	$2.870979 \times 10^{-20} \text{ J}$	$2.870979 \times 10^{-20} \text{ J}$	0
100	$2.870979 \times 10^{-19} \text{ J}$	$2.870979 \times 10^{-19} \text{ J}$	0
1000	$2.870979 \times 10^{-18} \text{ J}$	$2.870979 \times 10^{-18} \text{ J}$	0

Observation 8.2

The net is exactly zero for every N . The demon can extract work; but so long as its memory is finite, it must eventually erase. At that moment everything extracted is returned.

Observation 8.3

With an infinite memory the demon could extract work forever. What breaks in that case is not the second law but the assumption of finiteness: as in Paper 136, where an exact equality hides in a tail, one must say which assumption is doing the work before saying what was broken.

9. Separator and fences

Separator: which operation the cost is assigned to. Measurement, the physical act of looking, erasure, the reservation of room, or the clearing of it. **And when it is paid.**

Fences:

1. The debate is not declared settled: generalisations of the second law including measurement are still being studied.
2. It is not claimed that the second law is violated, nor that it is not.
3. Nothing is said about real hardware: truth tables were counted.
4. No theory of feedback control is built.
5. None of the five is called right: only that they placed the cost differently.

10. Roll call and closing

1. One bit costs 2.870979×10^{-21} J (Check 3.1): **0.017919 eV**.
2. The Szilard engine returns exactly that (Check 4.1): **a difference of 6.5×10^{-35} J**.
3. AND loses **1.188722 bits** and XOR exactly **1.000000** (Section 5).
4. The separator is the bias of the output (Observation 5.2): **0.811278** against **1.000000**.
5. Toffoli and Fredkin lose **0 bits** (Check 6.1): **and both are involutions**.
6. A reversible AND loses **0 bits** and needs a **third wire** (Section 7).
7. The net is exactly zero for every N (Check 8.1).

Closing: the demon's cost was not one place. Maxwell placed none, Szilard placed it on measurement, Brillouin on the act of looking, Landauer on erasure, and Bennett on erasure alone. All five were trying to save the second law, and what differed was only which operation they named. And once numbers are put in, an irreversible gate pays one bit turns out to be wrong as well: AND pays 1.188722 bits, and only erasure and XOR pay exactly one. A sixth place then appeared: reversible computation did not remove the cost but turned it into room and postponed it. It is paid later.

Revision Record

Second edition (September 2026): no claim, theorem or check has been altered from the first edition. In the English references, the lines citing earlier papers of this corpus still carried their Japanese titles; **they now carry the English titles.** **Nothing of substance has changed, and nothing has been withdrawn.**

The dashes used as separators in the text were replaced by punctuation (October 2026). No claim, number or reference was changed.

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