

The Four Pi That Catches Molecules Is the Four Pi That Spreads a Field

— Steady Diffusion Is Laplace's Equation: the Flux Through Every Enclosing Sphere Is 12.566370614359, and
in Two Dimensions No Steady Absorber Exists — [Paper 249]

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— *From thinking to syncing.* —

Abstract

When 4π turns up in a biological setting, is it a coincidence of numbers or the same root? This paper treats exactly one case: the 4π in the rate at which a cell captures molecules is the same solid angle as in Gauss's law. Not by analogy, but because it is the same equation. No new mathematical theorem and no new law is claimed. First, the flux is the same through every enclosing sphere. Moving r/a from 1.5 to 1000 gives **12.566370614359** throughout, the six values spreading by 3.55×10^{-15} (Section 3). Second, solving numerically without inserting the formula gives the same value. With the outer boundary at $R/a = 400$ the result is 12.597865194, and the gap comes from the boundary and not from the method: the ratio is **1.002506259**, agreeing with $R/(R - a) = 1.002506266$ to 6.7×10^{-9} (Section 4). Third, this is the core. Capture is proportional to the radius and not to the area. Ten times the radius gives a hundred times the surface and only ten times the catch: capture per unit area falls from 10 to **0.1** (Section 5). Fourth, in two dimensions no steady absorber exists. The flux falls as one over the logarithm of R/a , and even at $R/a = 10^{100}$ only **0.004342945** remains (Section 6). Fifth, the surface may be left almost bare. With receptors of relative radius 10^{-3} , **3142** of them reach half the rate of a perfect absorber: covering **0.0785** percent of the surface (Section 7). Sixth, three kinds of π appear in this neighbourhood: a solid angle, a convention of units, and one that is not 4π at all (Section 8).

Scope of this paper (scope note) : No new mathematical theorem and no new law is claimed: the Smoluchowski capture rate, the receptor count of Berg and Purcell, and the reduction of steady diffusion to Laplace's equation are all standard. No measured value is cited: every number is computed from a definition, and no measurement on a real cell is used. No physiology is discussed: nothing is claimed about the actual size, number or arrangement of receptors. The ratio 10^{-3} in Section 7 is a value put there for the sake of the calculation and belongs to no particular cell. No biological conclusion is drawn: nothing is said about why cells are the size they are or about how evolution acted. The reaction-limited case is not treated: only the diffusion-limited steady state appears, with no binding or unbinding rates. No general account of 4π in biology is attempted: one case is treated. Joining things because their numbers agree is what this paper most wants to avoid: the claim of a shared root rests on the equation being the same, not on the value being the same. Relation to earlier papers: Paper 2 treated the pure solid angle appearing in an inverse-square field. This paper shows the same solid angle in diffusion, from the identity of the equations. Paper 1 established $n = d - 1$: the failure in two dimensions in Section 6 is where that exponent becomes zero. Paper 156 traced the flatness inside a spherical shell to harmonicity. This paper uses the same harmonicity. Papers 36 and 37 dissected the 6π of Stokes drag: Section 8 places that 6π beside this 4π as a different root. Paper 44 showed that the power of 4π is a mass dimension and that fixing it is a convention of electromagnetic units: Section 8 puts that conventional 4π third. What is added is a check of the shared root through the radius independence of the flux, a numerical demonstration that capture scales with radius, a count of the two-dimensional failure as one over a logarithm, and a separation of the π s in this neighbourhood into three kinds.

1. Notation and placement

Setting: let a sphere of radius a absorb on contact the molecules drifting around it. With a distant concentration C_0 and a diffusion coefficient D , ask how much it captures per unit time in **the steady state**. The answer is $J = 4\pi DaC_0$ (Smoluchowski, 1917).

The question is what that 4π is. A coincidence of numbers, or the same object as the 4π of Gauss's law?

The position taken here is that it is the same. Not because the values agree: but because it comes from the same equation.

No biological conclusion is drawn and no measurement on a cell is used. Only the diffusion-limited steady state is treated.

2. Steady diffusion is Laplace's equation

The diffusion equation relates the change of concentration in time to D times its Laplacian. In the steady state the time derivative vanishes and what remains is the vanishing of the Laplacian, and nothing else.

Observation 2.1

This is the same equation as electrostatics. The potential where there is no charge and the concentration in a steady state satisfy one Laplace equation. The solutions therefore take the same form, and in spherical symmetry $1/r$ appears.

Observation 2.2

The 4π enters when the flux through a sphere is counted. It is exactly the reason 4π appears in Gauss's law: the full solid angle of a sphere is 4π .

Observation 2.3

Paper 156 traced the flatness inside a spherical shell to harmonicity. What this paper uses is that same harmonicity: diffusion, electrostatics and the interior of a shell are three faces of one property.

3. Check: the flux is the same through every sphere

Setting: put $D = C_0 = a = 1$ and measure the flux through a sphere of radius r , from the steady solution $C(r) = C_0(1 - a/r)$. If Gauss's law holds, moving r must not move the value.

r/a	flux through that sphere
1.0	12.566370614359172
1.5	12.566370614359172
2.0	12.566370614359172
10	12.566370614359172
100	12.566370614359174
1000	12.566370614359171

The six values spread by 3.55×10^{-15} , and $4\pi = 12.566370614359172$.

Observation 3.1

That the flux does not depend on the radius is Gauss's law itself. It has the same shape as the electric flux being equal through every surface enclosing a charge: and it is not a resemblance: it is the same consequence of the same equation.

Observation 3.2

The 4π of $J = 4\pi DaC_0$ is therefore a solid angle. It is the same object as the pure solid angle Paper 2 found in an inverse-square field.

4. The same value without the formula, and where the small gap comes from

Check 4.1: without using the form of the solution, integrate the radial equation in d dimensions over 2000001 points, placing the outer boundary at $R/a = 400$ and at 4000.

d	R/a	numerical	finite R formula	R infinite
3	400	12.597865194	12.597865278	12.566370614
3	4000	12.569504615	12.569512993	12.566370614
4	400	39.478663560	39.478664346	39.478417604
5	400	78.956833300	78.956836442	78.956835209

Observation 4.2

The numerical value differs from the infinite formula by 3.1×10^{-2} . That is not an error: the ratio is **1.002506259**, agreeing with $R/(R - a) = 1.002506266$ to 6.7×10^{-9} . What differs is not the method but the outer boundary being finite.

Observation 4.3

Discipline: when a number and a formula disagree, first ask what each of them assumed. Here the formula assumed an infinite outer boundary while the computation used $R = 400$: stretching R to 4000 shrinks the gap from 3.1×10^{-2} to 3.1×10^{-3} , exactly a tenth.

5. The core: capture scales with the radius, not with the area

Check 5.1: vary the radius and place capture beside surface area.

a	capture $4\pi DaC_0$	surface $4\pi a^2$	capture per unit area
0.1	1.256637061	0.125663706	10.000000000
0.5	6.283185307	3.141592654	2.000000000
1.0	12.566370614	12.566370614	1.000000000
5.0	62.831853072	314.159265359	0.200000000
10.0	125.663706144	1256.637061436	0.100000000

Observation 5.2

Ten times the radius gives a hundred times the surface. The catch grows only tenfold: and capture per unit area falls from 10 to 0.1, by a factor of a hundred.

Observation 5.3

One naturally expects a wider absorbing surface to catch more. That is surface-limited thinking. In the diffusion-limited case what decides is not the surface but the rate at which molecules arrive from far away, which is proportional to the radius: a gradient of $1/r$ multiplied by an area r^2 leaves one power of r .

Observation 5.4

That multiplication of $1/r$ by r^2 is exactly the $n = d - 1$ of Paper 1. One exponent is at work in the field, in the shell and in diffusion alike.

6. In two dimensions no steady absorber exists

Check 6.1: the same calculation in d dimensions gives a solution containing r to the power $2 - d$. At $d = 2$ that power is zero, the power-law solution disappears and a logarithm takes its place. Measure the flux with the outer boundary at R .

R/a	flux divided by $2\pi DC_0$
10	0.434294482
10^3	0.144764827
10^6	0.072382414
10^{12}	0.036191207
10^{100}	0.004342945

Observation 6.2

The flux falls as one over the logarithm of R/a . Even stretched to 10^{100} , 0.004342945 remains, and it reaches zero only in the limit. Yet in that limit the steady solution itself does not exist.

Observation 6.3

A creature in a flat world cannot gather food by diffusion alone. More precisely, it needs a wall at a finite distance. It captures while R is finite and stops when R is sent to infinity.

Observation 6.4

This is where the $n = d - 1$ of Paper 1 becomes 1 at $d = 2$ and a logarithm replaces $1/r$. It is a statement about that exponent, and not about parity or symmetry.

7. Berg and Purcell: the surface may be left almost bare

Setting: the whole surface need not absorb. With N small receptors of radius s scattered over the sphere, the capture rate is $Ns/(Ns + \pi a)$ times that of a perfect absorber (Berg and Purcell, 1977).

Check 7.1: put the relative radius at $s/a = 10^{-3}$ and compute the number of receptors needed for a target rate, together with the fraction of surface they cover.

target	receptors	fraction of surface covered
half of a perfect absorber	3142	0.078540 percent
ninety percent	28275	0.706858 percent
ninety-nine percent	311018	7.775442 percent

Observation 7.2

Zero point zero eight percent of the surface gives half the rate of a perfect absorber. More than ninety-nine percent may be left bare, because a molecule strikes the surface many times by diffusion. Missing once, it wanders off and comes back.

Observation 7.3

The ratio 10^{-3} is a value put there for the calculation. It is not a measurement on any cell. Nothing is claimed about the actual size, number or arrangement of receptors.

8. A roll call of the pi in this neighbourhood

Several π s appear around this subject. There are three kinds.

where it appears	what that π is
diffusion capture $4\pi DaC_0$	**a solid angle** (this paper)
the volume of a sphere	**a solid angle**
Gauss's law	**a solid angle**
Stokes drag $6\pi\eta a$	the π is a solid angle but **the 6 has another source** (Papers 36 and 37)
the Einstein relation $D = k_B T / (6\pi\eta a)$	**inherited from the 6π of Stokes**
the 4π in Poisson's equation in Gaussian units	**a convention of units** (Paper 44)

Observation 8.1

The same characters 4π sometimes mean a solid angle and sometimes a convention of units. And 6π is not one and a half times 4π : as Papers 36 and 37 showed, the 6 has a different origin.

Observation 8.2

The only case in which this paper claims a shared root is diffusion capture. The claim rests on the same Laplace equation, not on the values agreeing. Joining things because their numbers agree is what this paper most wants to avoid.

9. Separator and fences

Separator: whether it comes from the same equation. That two values agree shows nothing about a shared root. **The way to check is to move something:** here the radius was moved and the flux did not move.

Fences:

1. **No physiology is discussed: nothing is claimed about actual receptors.**
2. **No measured value is used: everything is computed from definitions.**
3. **Nothing is said about cell size or evolution.**
4. **The reaction-limited case is not treated: only the diffusion-limited steady state.**
5. **No general account of 4π in biology is attempted: one case was treated.**

6. Nothing is joined on the ground that its numbers agree.

10. Roll call and closing

1. The flux does not depend on the radius (Check 3.1): **12.566370614359**, spreading by 3.55×10^{-15} .
2. The same value appears numerically (Check 4.1): the ratio **1.002506259** matches $R/(R - a)$.
3. Capture scales with radius, not area (Check 5.1): per unit area from 10 to 0.1.
4. No steady absorber in two dimensions (Check 6.1): **0.004342945** even at $R/a = 10^{100}$.
5. Half the rate from 0.0785 percent of the surface (Check 7.1): **3142** receptors.
6. Three kinds of π nearby (Section 8): solid angle, convention, and not 4π at all.

Closing: the 4π in the rate at which a cell captures molecules is the same solid angle as in Gauss's law. Not because the values agree, but because steady diffusion is Laplace's equation. The same equation gives the same $1/r$, multiplied by the same spherical area, leaving the same 4π . And the check is that the flux does not depend on the radius. Two things followed: capture scales with the radius and not the area, and no steady absorber exists in two dimensions. Last, the surface may be left almost bare. Zero point zero eight percent gives half the rate, because a molecule, by diffusion, comes back again and again.

Revision Record

Second edition (September 2026): no claim, theorem or check has been altered from the first edition. In the English references, the lines citing earlier papers of this corpus still carried their Japanese titles; **they now carry the English titles**. The marks for caution and emphasis that remained in the body text have also been removed. **Nothing of substance has changed, and nothing has been withdrawn.**

The dashes used as separators in the text were replaced by punctuation (October 2026). No claim, number or reference was changed.

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