

Time Is One-Dimensional Because Prediction Demands It, Not Because a Law Says So

— The Same Procedure, With Only the Signature Changed, Gives Amplifications of **0.012** and 3.5×10^{32} —

[Paper 253]

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— *From thinking to syncing.* —

ABSTRACT

Why is time one-dimensional? The answer offered here is that no law decrees it; the requirement that one be able to predict allows nothing else. No new mathematical theorem and no new law is claimed. First, being solvable and being predictable are different demands. Of Hadamard's three conditions, existence and uniqueness concern solvability, and only the third—continuous dependence on the initial data—corresponds to prediction (Section 2). Second, with 0 time dimensions the initial value problem explodes. Solving Laplace's equation as an initial value problem, initial data of size $1/n$ is amplified at $y = 1$ by 1.101×10^3 at $n = 10$ and by 3.463×10^{32} at $n = 80$ —the smaller the data, the larger the solution (Section 3). Third, with 1 time dimension the same procedure stays bounded. The wave equation with the same data amplifies by 0.054 at $n = 10$ and **0.012** at $n = 80$ —at the same $n = 80$, a difference of **34** orders of magnitude (Section 4). Fourth, this is the core. One and the same heat equation exchanges well-posedness for ill-posedness when only its direction is reversed. Mode $n = 80$ is multiplied by 1.604×10^{-28} forwards and by 6.235×10^{27} backwards—relative noise of 10^{-10} has grown, backwards, to 4.135×10^{17} already at $t = 0.001$ (Section 5). Fifth, with 2 time dimensions what happens next is not determined. The dispersion relation $\omega_1^2 + \omega_2^2 = |k|^2$ is solved by an entire circle in the ω plane—one time dimension gives the two values $\omega = \pm |k|$ (Section 6). Sixth, one measure separates them: the signature of the principal symbol. $(4, 0)$ is elliptic, $(3, 1)$ hyperbolic, $(2, 2)$ ultrahyperbolic—and the initial value problem is well-posed for $\mathbf{q} = \mathbf{1}$ alone (Section 7).

Scope of this paper (scope note) : No new mathematical theorem and no new law is claimed—Hadamard's three conditions, the ill-posedness of the Cauchy problem for Laplace's equation, the ill-posedness of the backward heat equation, the Cauchy problem for ultrahyperbolic equations, and the classification of second order operators by signature are all standard. It is not proved that spacetime must be $3+1$ —the discipline is that of Paper 69, and what is done here is to add one entry to its ledger. No anthropic argument is made—the phrase because there are observers is never used. No measured value is cited—every number is computed from a definition. No theory of partial differential equations is built—only exact solutions and a finite Fourier representation are used. Prediction is not defined—what is treated is the single point of continuous dependence on the data, and no epistemology is entered. It is not claimed that ultrahyperbolic equations have no solutions—solutions exist; what fails is uniqueness and continuous dependence. The arrow of time is not solved—what Paper 98 recorded as open is left untouched. Section 5 says only that one equation differs in well-posedness between its two directions, and nothing about why there is a low entropy past. Quantum theory is not treated. The relation to earlier papers. Paper 69 assembled five independent roots selecting $D = 4$ —but not one of them selects which of the four is time. This paper fills that empty place. Paper 159 showed that ill-posed is not one word and placed the separator in the decay of the singular values—this paper applies that same separator to the number of time dimensions. Paper 154 showed that a limit without its order is not a quantity—that concerns the order of limits and this the signature; the axes differ. Papers 118, 119 and 122 showed that the one word time covers four logical types—what is asked here is not the type but the number. Paper 251 separated special in two dimensions—this paper stands beside it, on the side of special in one. What is added is tabulating the amplification against n so that the 34 orders of magnitude can be read off, running one and the same heat equation in both directions and measuring the noise component, recording that the ultrahyperbolic dispersion relation is solved by a whole circle, and adding one independent root to the census of Paper 69.

1. Notation and standing

The setting: over p dimensions of space and q of time, consider a second order linear equation. **The signature of its principal symbol is (p, q) .** For $q = 0$ it is elliptic, for $q = 1$ hyperbolic, and for $q \geq 2$ ultrahyperbolic.

The question: why is time one-dimensional? Can the equations of a universe with two times not be written down?

They can. They can be written, and tomorrow is not determined. This paper puts numbers on the difference.

It is not proved that spacetime must be $3+1$ (Paper 69). What is done is only to separate the mechanisms that select $q = 1$ from those that do not.

2. Being solvable and being predictable are different demands

Hadamard wrote what it is for a problem to be well-posed as three conditions.

the demand	what happens when it fails
a solution exists	there is no answer
the solution is unique	there is no saying which one is the answer
it depends continuously on the data	a difference too small to measure changes the answer

Observation 2.1

The three are independent. Existence and uniqueness are about whether it can be solved; what corresponds to prediction is the third alone.

Observation 2.2

Why the third is prediction. Initial data is measured and then entered, and measurement always carries error. If the error changes the answer, then holding a formula for the solution, one still cannot state tomorrow's value.

Observation 2.3

Paper 159 showed that ill-posed is not one word and placed the separator in the decay of the singular values —a power law can be inverted, an exponential cannot. This paper applies that separator to the number of times.

3. Check: with no time dimension the initial value problem explodes

Check 3.1: take Laplace's equation $u_{xx} + u_{yy} = 0$ and, treating y as the time, solve it as an initial value problem with $u(x, 0) = 0$ and $u_y(x, 0) = \sin(nx)/n$. The exact solution is $u = \sin(nx) \sinh(ny)/n^2$ (Hadamard's example).

n	size of the data	solution at $y = 1$	amplification
10	1.000×10^{-1}	1.101×10^2	1.101×10^3
20	5.000×10^{-2}	6.065×10^5	1.213×10^7
40	2.500×10^{-2}	7.356×10^{13}	2.942×10^{15}
80	1.250×10^{-2}	4.329×10^{30}	3.463×10^{32}

Observation 3.2

The data tends to zero and the solution diverges. Raising n makes the data as small as one likes, and each time the solution grows exponentially.

Observation 3.3

Neither existence nor uniqueness has failed. There is a solution and it is unique—what has failed is the third condition alone. The gap between solvable and predictable appears here in its bare form.

4. Check: with one time dimension the same procedure stays bounded

Check 4.1: change the equation to the wave equation $u_{tt} = u_{xx}$. One sign has been changed, and nothing else. With the same data $u_t(x, 0) = \sin(nx)/n$ the exact solution is $u = \sin(nx) \sin(nt)/n^2$.

n	size of the data	solution at $t = 1$	amplification
10	1.000×10^{-1}	5.440×10^{-3}	0.0544
20	5.000×10^{-2}	2.282×10^{-3}	0.0457
40	2.500×10^{-2}	4.657×10^{-4}	0.0186
80	1.250×10^{-2}	1.553×10^{-4}	0.0124

Observation 4.2

A $\sinh$ became a $\sin$. That is the whole of it. At the same $n = 80$ the amplification is 3.463×10^{32} against 0.0124——34 orders of magnitude.

Observation 4.3

What changed is one sign in the equation, and not the data, not the method, not the precision. What makes the difference is the signature.

5. The core: one equation exchanges well-posedness with its direction

The setting: take the heat equation $u_t = u_{xx}$. Mode n is multiplied by $e^{-n^2 t}$. Advancing t is forwards and retreating is backwards. **There is only one equation; only the direction is changed.**

n	forwards, $t = +0.01$	backwards, $t = -0.01$
10	3.679×10^{-1}	2.718
20	1.832×10^{-2}	5.460×10^1
40	1.125×10^{-7}	8.886×10^6
80	1.604×10^{-28}	6.235×10^{27}

Check 5.1: run it. A Gaussian bump is placed on a grid of 512 points and sent both ways in a Fourier representation.

time	maximum, forwards	maximum, backwards
0	1.000000	1.000000
0.001	0.975900	2.073067×10^{11}
0.005	0.894427	2.416894×10^{123}
0.010	0.816497	1.304268×10^{264}

Forwards it stays below one. Backwards it overflows double precision before reaching $t = 0.05$.

Check 5.2: what this means for prediction. **Relative noise of 10^{-10}** —standing for observational error—is added to the data and that component alone is sent both ways.

time	noise component, forwards	noise component, backwards
0.001	9.398×10^{-11}	4.135×10^{17}
0.005	5.403×10^{-11}	9.713×10^{130}
0.010	3.835×10^{-11}	1.643×10^{273}

Observation 5.3

Forwards the noise decays. Backwards it is 10^{17} at $t = 0.001$. The signal is buried—one holds the same formula for the solution and cannot state a value.

Observation 5.4

This is the core. There is no asymmetry on the side of the law. $u_t = u_{xx}$ is one equation, and $t \mapsto -t$ does not turn it into another. The asymmetry is on the side of well-posedness—one and the same equation permits prediction in one direction only.

Observation 5.5

This does not explain the arrow of time. As Paper 98 recorded honestly, why there is a low entropy past is unsolved. What can be said here is one step short of that—the phenomenon of being able to predict one way and not the other does not itself require an asymmetric law.

6. With two time dimensions what happens next is not determined

The setting: give time two dimensions, $u_{t_1t_1} + u_{t_2t_2} = u_{x_1x_1} + u_{x_2x_2}$, signature $(2, 2)$. The equation is written. Solutions exist.

Check 6.1: a plane wave $e^{i(k \cdot x - \omega \cdot t)}$ gives the dispersion relation $\omega_1^2 + \omega_2^2 = |k|^2$.

$ k $	sample solutions (ω_1, ω_2)	the solution set
1	(1, 0), (0.7071, 0.7071), (0, 1)	the whole circle of radius 1
2	(2, 0), (1.4142, 1.4142), (0, 2)	the whole circle of radius 2
5	(5, 0), (3.5355, 3.5355), (0, 5)	the whole circle of radius 5

Observation 6.2

With one time the same procedure returns two values, $\omega = \pm |k|$. With two it returns a continuum.

Observation 6.3

So the data does not determine what happens next. What has failed this time is uniqueness—in Section 3 it was continuous dependence. One word, ill-posed, is naming two different failures (Paper 159).

7. One measure: the signature of the principal symbol

Check 7.1: count the signs of the eigenvalues of the principal symbol. p counts the positive and q the negative.

signature (p, q)	q , the number of times	type	initial value problem	example
(4, 0)	0	elliptic	ill-posed	Laplace
(3, 1)	1	hyperbolic	well – posed	wave, Maxwell
(2, 2)	2	ultrahyperbolic	ill-posed	—
(1, 3)	3	ultrahyperbolic	ill-posed	—

Observation 7.2

Only $q = 1$ is well-posed. (1, 3) is (3, 1) with the overall sign reversed and means the same physics—what is to be counted is the size of the smaller sign class.

Observation 7.3

It is not that time is special. The sign class with only one member is what we call time. The direction looks reversed because the definition comes first and time second.

Observation 7.4

Papers 118, 119 and 122 showed that the one word time covers four logical types. What is asked here is not the type but the number, a different axis—all four clocks ride on $q = 1$.

8. One independent entry for the census of Paper 69

Paper 69 wrote why $3+1$ as an overdetermination—several independent criteria all pointing at $D = 4$.

the root assembled in Paper 69	what it selects	does it select the number of times
conformal invariance of the Maxwell action	$D = 4$	no
graviton degrees of freedom $\frac{1}{2}D(D - 3)$	$D = 4$	no
exotic $\mathbb{R}^4$ (Paper 105)	$D = 4$	no
Bertrand and Ehrenfest stability	$d = 3$	no
the maximum of the ball volume (Paper 59)	$d = 5$	no
well – posedness(thispaper)	$q = 1$	yes

Observation 8.1

All five of Paper 69's roots select how many. Not one of them selects which of them is time. That is the empty place this paper fills.

Observation 8.2

And this root cannot be derived from the other five. Conformal invariance, graviton degrees of freedom and exotic $\mathbb{R}^4$ are all statements made after a signature has been assumed——well-posedness stands on the side of that assumption. They are distinct roots (Paper 58).

Observation 8.3

Even so, 3+1 is not proved here. What can be said reaches no further than that prediction is not an available activity unless $q = 1$, and adds nothing about why $p = 3$. No anthropic argument is made——the phrase because there are observers is never used.

9. Roll call and closing

1. Solvable and predictable differ (Observation 2.1): of Hadamard's three, prediction is the third alone.
2. No time dimension explodes (Check 3.1): amplification 3.463×10^{32} at $n = 80$.
3. One time dimension stays bounded (Check 4.1): 0.0124 at the same $n = 80$, 34 orders apart.
4. One equation, two directions (Check 5.1): 0.816 forwards against 1.304×10^{264} backwards.
5. The noise says it again (Check 5.2): 10^{-10} becomes 4.135×10^{17} at $t = 0.001$.
6. Two time dimensions are not unique (Check 6.1): the solutions fill a circle.
7. The signature separates them (Observation 7.2): only $q = 1$ is well-posed.
8. An independent entry for Paper 69 (Observation 8.2): the other five speak after a signature is assumed.

Closing: time is one-dimensional, and not because time is special. The equations of a two-time universe can be written, and their solutions exist. What breaks is prediction——add a dimension and the answer stops being unique; remove one and measurement error buries it. There is exactly one signature in which the word tomorrow means anything. And Section 5 showed that even inside that one, one direction permits prediction and the other does not——without once invoking an asymmetric law.

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