

# The Only Scale at Which Dynamic Similarity Is Exact Is One

— Once There Are Two  $\Pi$  Groups, the Fluid the Model Needs Does Not Exist — Ships Split the Resistance and  
Biology Runs Out of Range — [Paper 256]

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— *From thinking to syncing.* —

## Abstract

Paper 117 showed that the condition under which dimensional analysis fixes an exponent is that there be exactly one  $\Pi$  group, and that adding viscosity alone breaks it. This paper asks what happens outside that fence—the answer is that the model can no longer be built. Shrinking the geometry similarly removes not one of the  $\Pi$  groups, and with two of them there is no solution other than a scale of 1. No new mathematical theorem and no new law is claimed. First, the condition of 117 is applied to a ship. A flow with no free surface has **1** group, and a ship, where gravity enters, has **2**—the fence that 117 broke with viscosity is broken here by gravity (Section 2). Second, this is the core of the paper. Matching the Froude number at a scale of  $1/25$  demands a kinematic viscosity of  $8.0 \times 10^{-9} \text{ m}^2/\text{s}$ . Water is **125.73** times larger, and mercury, the least viscous ordinary liquid at room temperature, is still **14.09** times larger—no such liquid exists (Section 3). Third, the condition for both to hold is  $\nu_m/\nu_s = \lambda^{3/2}$ . With the same fluid the only solution is  $\lambda = 1$ —the only scale at which dynamic similarity is exact is full size (Section 3). Fourth, the size of the departure is put into numbers. The friction coefficient of a  $1/25$  model matched only in Froude number is **2.0382** times that of the ship—which is why ship models split the resistance in two, as a procedure and not as a theory (Section 4). Fifth, the same figure appears in biology. Geometry gives a metabolic exponent of  $2/3$  while measurement leans towards  $3/4$ . Even across a mass ratio of  $2 \times 10^5$  the two predictions separate by only **2.7654** (Section 5). Sixth, the separation is  $\mathbf{R}^{1/12}$ . Seeing a factor of **2** requires a mass range of **4096**—over a narrow range the two exponents cannot be told apart (Section 6).

Scope of this paper (scope note): No new mathematical theorem and no new law is claimed—the Buckingham Pi theorem, the definitions of the Reynolds and Froude numbers, the splitting of resistance under Froude's hypothesis, and Kleiber's law are all standard. Neither the counting of  $\Pi$  groups nor the condition under which they fix an exponent is a contribution of this paper—Paper 117 already wrote the condition that there be exactly one  $\Pi$  group, and showed that adding viscosity alone breaks it. This paper receives that condition as a premise and treats only what happens on the broken side. No fluid dynamics is built—what is used is a count of variables, two ratios, and one empirical line. The existence of solutions of Navier-Stokes is not discussed—Paper 03 treats that. What this paper receives from 03 is the single point that the dimensionless parameter of a flow with no free surface reduces to one alone. Self-similarity and renormalisation are not treated—the similarity of Papers 117, 94 and 144 is self-similarity, a different notion from the dynamic similarity of this paper. Anomalous dimensions are not treated—the counterexample of 117 lies there. This paper stays on the side of building a model in a laboratory. The metabolic exponent is not settled here—it is not claimed that  $3/4$  is right, nor that  $2/3$  is. What this paper counts is the mass range required to tell the two apart. No claim about biology is made—no transport-network theory is adopted or rejected. No empirical line is used as a theorem—the friction formula in Section 4 is a correlation line and not a theorem. It is used only to see the order of the departure, and the value 2.0382 is not itself claimed. Relation to earlier papers: Paper 117 separated the four ways in which scale invariance fixes an exponent and wrote the condition that there be exactly one  $\Pi$  group—this paper stands outside that condition. 117 wrote that the exponent stops being determined; this paper writes that the model stops being buildable—two different ways of breaking the same fence. Paper 03 wrote that the dimensionless parameter reduces to the Reynolds number alone—this paper treats the case with one gravity added. Paper 94 showed that dimension is not one thing—this paper adds one line, that similarity is likewise not one thing, dynamic and self-similarity being distinct. Paper 144 read the exponent as the signature of what is conserved—this paper puts a number on the range required for that signature to be legible. Paper 190 measured rare on a logarithmic scale—this paper likewise writes the gap between two exponents not as a ratio but as the range over which they can be distinguished. What is added is applying the condition of 117 to a ship, where the breaking variable is gravity rather than viscosity, setting the required kinematic viscosity of  $8.0 \times 10^{-9}$  against real liquids and obtaining a shortfall of 14.09 even for mercury, stating that the joint condition admits  $\lambda = 1$  alone, putting the departure into a number as a friction coefficient ratio of 2.0382, and obtaining a required mass range of 4096 for telling  $2/3$  from  $3/4$ , and identifying the separation as  $R^{1/12}$ .

## 1. Notation and Placement

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**Setting:** write the ratio of lengths between model and full size as  $\lambda = L_m/L_s$ . **Geometric similarity** means that the shapes are similar; **dynamic similarity** means that every dimensionless number agrees.

**What is received:** Paper 117 showed that the condition for dimensional analysis to fix an exponent is that there be exactly one  $\Pi$  group, and that adding viscosity alone breaks it. This paper receives that condition as a premise—neither the counting nor the condition is a contribution here.

**Question:** outside that fence, on the side where there are two  $\Pi$  groups, what happens when one actually tries to build a model?

The answer is that it cannot be built. Shrinking the geometry similarly removes not one of the  $\Pi$  groups, and with two of them there is no solution other than  $\lambda = 1$ .

A caution: the similarity of this paper is dynamic similarity. It is a different notion from the self-similarity of Papers 117, 94 and 144—the two are not to be conflated.

2. Applying the Condition of 117 to a Ship

**Setting:** the Buckingham Pi theorem says that the number of independent dimensionless groups is the number of variables minus the number of dimensions. 117 broke the fence here by adding viscosity—this paper adds gravity.

**Check 2.1:** the dimensions are the three  $M$ ,  $L$  and  $T$ .

| Case                      | Variables            | $\Pi$ groups | Content                 |
|---------------------------|----------------------|--------------|-------------------------|
| Flow with no free surface | $V, L, \rho, \mu$    | $4 - 3 = 1$  | <b>Re</b> alone         |
| Ship with a free surface  | $V, L, \rho, \mu, g$ | $5 - 3 = 2$  | <b>Re</b> and <b>Fr</b> |

Observation 2.2

what was added is the single gravitational acceleration  $g$ . One extra variable took the  $\Pi$  groups from 1 to 2—the same figure as the breaking that 117 exhibited with viscosity.

Observation 2.3

Paper 03 wrote that the dimensionless parameter of Navier-Stokes reduces to the Reynolds number alone. That is the first row of the table above—what this paper treats is the second.

Observation 2.4

what is lost after the break differs between 117 and this paper. In 117 the exponent stops being determined—here the model stops being buildable. Two different ways of breaking the same fence.

### 3. The Core: The Required Fluid Does Not Exist

**Setting:** to shrink the waves of a free surface correctly one matches the Froude number  $\text{Fr} = V/\sqrt{gL}$ . Gravity cannot be shrunk, so  $g$  is shared, and the velocity ratio is thereby forced to  $V_m/V_s = \sqrt{\lambda}$ .

**Check 3.1:** the ratio of Reynolds numbers  $\text{Re} = VL/\nu$  is then  $\lambda \cdot \sqrt{\lambda} = \lambda^{3/2}$ .

| Scale $\lambda$ | Velocity ratio | Ratio of Re | Required $\nu$ [m <sup>2</sup> /s] |
|-----------------|----------------|-------------|------------------------------------|
| 1/5             | 0.447214       | 0.08944272  | $8.9443 \times 10^{-8}$            |
| 1/10            | 0.316228       | 0.03162278  | $3.1623 \times 10^{-8}$            |
| 1/25            | 0.200000       | 0.00800000  | $8.0000 \times 10^{-9}$            |
| 1/100           | 0.100000       | 0.00100000  | $1.0000 \times 10^{-9}$            |

#### Observation 3.2

the velocity can be lowered only as  $\sqrt{\lambda}$ , so Re falls as  $\lambda^{3/2}$ . The target is a ratio of 1, so the only way to close the gap is to change the fluid.

**Check 3.3:** set the  $8.0 \times 10^{-9} \text{ m}^2/\text{s}$  required at a scale of 1/25 against real liquids.

| Fluid                 | $\nu$ [m <sup>2</sup> /s] | Times the required value |
|-----------------------|---------------------------|--------------------------|
| Water (20°C)          | $1.0058 \times 10^{-6}$   | 125.73                   |
| Acetone (20°C)        | $3.9031 \times 10^{-7}$   | 48.79                    |
| Liquid sodium (400°C) | $3.2710 \times 10^{-7}$   | 40.89                    |
| Mercury (20°C)        | $1.1275 \times 10^{-7}$   | 14.09                    |

#### Observation 3.4

the least viscous ordinary liquid at room temperature is mercury. Even mercury is 14.09 times too large—this is not a matter of looking for a better fluid, but of being short by an order of magnitude.

### Observation 3.5

written as a formula, the joint condition is  $\nu_m/\nu_s = \lambda^{3/2}$ . With the same fluid the left side is 1, so  $\lambda^{3/2} = 1$ , that is  $\lambda = 1$ —the only scale at which dynamic similarity is exact is full size.

### Observation 3.6

the separator—what has to be changed for these to become different things. The number of  $\Pi$  groups. With one a model can be built; with two it cannot. Making the shape similar removes not one of them—geometric similarity contributes nothing at all to dynamic similarity.

## 4. Putting the Departure into Numbers

**Setting:** how differently does a model behave when **Re** has not been matched? **Look through the friction correlation line**  $C_f = 0.075/(\log_{10} \text{Re} - 2)^2$ .

**Check 4.1:** the ship has **Re** =  $1.0 \times 10^9$  and the 1/25 model has **Re** =  $8.0 \times 10^6$ .

|            | Re                  | $C_f$         |
|------------|---------------------|---------------|
| Ship       | $1.000 \times 10^9$ | 0.001531      |
| 1/25 model | $8.000 \times 10^6$ | 0.003120      |
| Ratio      | —                   | <b>2.0382</b> |

### Observation 4.2

the friction coefficient of the model is 2.0382 times that of the ship. The shape is similar and the waves are matched, and yet the friction alone is out by a factor of 2—it is not similar.

### Observation 4.3

this is why ship models split the resistance into wave-making and friction. The wave side shrinks similarly under Froude, while the friction side is computed separately from the formula above and added back—this is a procedure to cope with being unable to match two at once, not a theory.

### Observation 4.4

a point to state honestly—the formula in Section 4 is a correlation line and not a theorem. It is used here to see the order of the departure, and the value 2.0382 is not itself claimed.

## 5. The Same Figure Appears in Biology

**Setting:** heat is shed by the surface ( $L^2$ ) and made by the volume ( $L^3$ ). Since mass goes as  $L^3$ , the surface is proportional to  $M^{2/3}$ —geometry alone gives an exponent of  $2/3$ . Measurement leans towards  $3/4$ .

**Check 5.1:** from a mouse (20 g) to an elephant (4000 kg) the mass ratio is  $2.000000 \times 10^5$ .

| Exponent | Origin                         | Ratio of metabolic rate |
|----------|--------------------------------|-------------------------|
| 0.6667   | Geometry (surface over volume) | 3420.0                  |
| 0.7500   | Kleiber (measurement)          | 9457.4                  |
| Ratio    | —                              | 2.7654                  |

### Observation 5.2

even across a mass range of  $2 \times 10^5$ , extreme for organisms, the two predictions separate by only 2.7654. The metabolic rates themselves, 3420 and 9457, are of the same order.

### Observation 5.3

the separation is exactly  $R^{1/12}$ —because  $3/4 - 2/3 = 1/12$ . That the exponents differ by only  $1/12$  decides everything that follows.

#### Observation 5.4

what the ship and the organism share is that there is more than one number to match. For the ship,  $R_e$  and  $Fr$ ; for the organism, the  $L^2$  of heat loss and the  $L^3$  of mass—neither pair can be met at a single scale.

## 6. Why a Century Has Not Settled It

**Check 6.1:** the mass range required to separate the two predictions by a factor  $k$  is  $R = k^{12}$ .

| Separation sought | Mass range required |
|-------------------|---------------------|
| 1.500             | $1.297 \times 10^2$ |
| 2.000             | $4.096 \times 10^3$ |
| 2.765             | $1.997 \times 10^5$ |
| 3.000             | $5.314 \times 10^5$ |

**Check 6.2:** conversely, the separation visible from the mass range in hand.

| Mass range | Separation of the two predictions |
|------------|-----------------------------------|
| $10^2$     | 1.4678                            |
| $10^3$     | 1.7783                            |
| $10^4$     | 2.1544                            |
| $10^5$     | 2.6102                            |
| $10^6$     | 3.1623                            |

#### Observation 6.3

a mass range of **4096** is needed merely to see a factor of 2. Such a range is hard to obtain within a single taxon, and over a narrow range the two exponents cannot be told apart—this is why the argument does not end.

**Observation 6.4**

report the range, not the ratio.“How many times more right is one than the other” is not an answer—the answer is what range one must take to tell them apart(the same practice as Paper 190).

**Observation 6.5**

Paper 144 read the exponent as the signature of what is conserved.What this section adds is the single point that legibility of that signature carries a condition on range—over a range of only  $10^3$ , two signatures differing by  $1/12$  look like the same character.

**7. Points to State Honestly**

**Check 7.1: the exponent itself is not settled.** The reported values are set side by side.

| Exponent | Origin                                  | Mouse to elephant ratio |
|----------|-----------------------------------------|-------------------------|
| 0.667    | Geometry                                | 3433.9                  |
| 0.686    | A report corrected for body temperature | 4330.2                  |
| 0.750    | Kleiber                                 | 9457.4                  |

**Observation 7.2**

that three values stand side by side is itself a consequence of Section 6.If the range available cannot separate a difference of  $1/12$ , then every one of these values will sit on the same data.

**Observation 7.3**

this paper says of no exponent that it is the right one.There are also reports that the log-log fit is not a straight line—if so, the metabolic exponent is not even a single number.

## 8. Roll Call and Closing

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1. The condition belongs to 117(Section 1): exactly one  $\Pi$  group; this paper stands outside it.
2. Here the breaker is gravity(Check 2.1):  $\Pi$  groups go from 1 to 2; 117 broke it with viscosity.
3. What is lost differs(Observation 2.4): 117 loses the exponent, this paper the model.
4. The required viscosity(Check 3.1):  $8.0 \times 10^{-9}$  at a scale of 1/25.
5. No such fluid exists(Check 3.3): even mercury is 14.09 times too large.
6. The only joint scale is full size(Observation 3.5):  $\nu_m/\nu_s = \lambda^{3/2}$  has  $\lambda = 1$  alone.
7. The departure is 2.0382(Check 4.1): 0.003120 against 0.001531.
8. Splitting the resistance is a procedure(Observation 4.3): it is not a theory.
9. Geometry gives 2/3, measurement 3/4(Check 5.1): 3420.0 and 9457.4, a ratio of 2.7654.
10. The separation is  $R^{1/12}$ (Observation 5.3): the exponents differ by only 1/12.
11. A factor of 2 costs a range of 4096(Check 6.1): over a narrow range they cannot be told apart.
12. The exponent is not settled(Check 7.1): 0.667, 0.686 and 0.750 stand together.

Closing: the fence that Paper 117 wrote was that there be exactly one  $\Pi$  group. What this paper has looked at is the country outside it—not only does the exponent stop being determined, but the model itself stops being buildable. Shrinking the geometry similarly removes not one of the  $\Pi$  groups, so with two of them there is no solution other than  $\lambda = 1$ . For the ship the required fluid falls short by 14.09 even in mercury, and the shortfall is patched by the procedure of splitting the resistance. For the organism the two exponents differ by only 1/12, and seeing a factor of 2 costs a range of 4096, so the matter is not settled. The figure is the same—there is more than one number to match. One thing separates them—counting the  $\Pi$  groups. Count them, and one knows whether a model can be built or whether a shortfall is being patched. Do not count them, and one goes on looking for a fluid that does not exist.

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